

Turbulent heat transfer in open-channel flows with a thermally conductive porous wall

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Direct numerical simulation (DNS) results of porous-wall turbulent flows in open channels with conjugate heat transfer are reported. For the conductive porous walls considered, the change in heat transfer is non-monotonic with permeability. The heat flux initially decreases when going from a conductive smooth wall to slightly porous walls. In this initial regime, the near-wall flow remains smooth-wall like and the heat transfer in the porous wall is dominated by molecular diffusion. A reduction of the more favourably conducting solid material then diminishes the heat transfer, whereas the skin friction monotonically increases with permeability. Beyond a certain permeability the near-wall flow transitions to the Kelvin–Helmholtz (K–H)-like regime (Gómez-de Segura & García-Mayoral 2019 *J. Fluid Mech.* 875, 124–172; Habibi Khorasani *et al.* 2024 *J. Fluid Mech.* 984, A63) with cross-stream rollers. The heat flux increases with permeability and eventually surpasses that of smooth-wall turbulence. Once in the K–H-like regime, turbulent heat transport can deeply penetrate the porous wall and be comparable to the turbulent heat transport in the channel region. Thermal performance is assessed in terms of the Reynolds analogy breakdown, which is the disparity between the fractional increases in the Stanton number, St , and the fractional increases in the skin-friction coefficient, C_f , relative to smooth-wall flow (Bunker 2013 *Proceedings of the ASME Turbo Expo 2013*; Rouhi *et al.* 2022 *J. Fluid Mech.* 951, A45). Similar to rough walls, the breakdown is unfavourable for porous walls, which is due to growing dissimilarities between the transfer of momentum and heat near the porous wall as it becomes more permeable. Turbulent sweep and ejection events

contribute more to momentum transfer than to heat transfer. However, unlike for rough walls, a saturation limit for heat transfer is not observed.

Key words: turbulent convection, turbulence simulation

1. Introduction

1.1. *Passive modification of momentum and heat transfer using surfaces*

Passive means of affecting turbulent flows for flow control purposes is an area of intense research. Different surfaces such as riblets, roughness and porous walls can alter the dynamics of flow turbulence (Gómez-de Segura & García-Mayoral 2019; Chung *et al.* 2021; Endrikat *et al.* 2021; Habibi Khorasani, Luhar & Bagheri 2024; Hao & García-Mayoral 2025). This, in turn, has motivated studies on the difference between turbulent heat transfer over such surfaces and smooth walls (Dipprey & Sabersky 1963; Webb, Eckert & Goldstein 1971; Han & Zhang 1992; García *et al.* 2012). Leonardi *et al.* (2015) investigated turbulent heat transfer over surfaces with transverse bars using direct numerical simulations (DNSs). They showed that the heat flux over the rough surfaces was higher than a smooth wall, although it was always accompanied by an even greater increase in drag. This difference in turbulent momentum and heat transfer was due to the flow separation occurring near the roughness elements, leading to increased contributions from pressure drag, which has no heat-transfer analogue (Owen & Thomson 1963).

Dissimilarity in the transport mechanisms of heat and momentum transfer means a breakdown of the Reynolds analogy (Von Kármán 1939), which can be assessed by the ratio $2St/C_f$, where St is the Stanton number and C_f the skin-friction coefficient. Deviations from unity represent a breakdown in the Reynolds analogy, which can be favourable (greater rate of increase in St relative to the rate of increase in C_f) or unfavourable (lower rate of increase in St relative to the rate of increase in C_f). For the bar-type roughness investigated by Leonardi *et al.* (2015), the breakdown is unfavourable.

Further investigations of rough-wall turbulent heat transfer were carried out by Peeters & Sandham (2019), who extended the conclusions of Leonardi *et al.* (2015) to irregularly rough surfaces. Their results indicated that heat transfer does not increase indefinitely as the size of the viscous-scaled roughness elements increases. This observation was also made by MacDonald, Hutchins & Chung (2019), who conducted minimal-channel DNSs of turbulent heat transfer over walls with sinusoidal roughness of different heights. They observed that, for a given physical roughness size, St achieves a maximum in the transitionally rough regime and subsequently decays monotonically in the form-drag-dominated fully rough regime.

Rouhi *et al.* (2022) gathered the heat-transfer efficiencies reported throughout the literature for both active and passive methods of affecting heat transfer (see figure 1 of their manuscript). Active methods such as impingement jets and wall blowing/suction demonstrated the greatest favourable performance. Rough surfaces, i.e. passive methods, consistently demonstrate unfavourable performance. Rouhi *et al.* (2022) also studied turbulent heat transfer over riblets. Most of the investigated riblets led to an unfavourable breakdown of the Reynolds analogy, with only certain triangular-shaped riblets achieving a locally favourable breakdown.

Much less is known about heat transfer over porous walls. Motoki *et al.* (2022) carried out DNSs of turbulent channel flow with heat transfer over a permeable wall. At higher permeability, a Kelvin–Helmholtz instability was triggered, significantly

enhancing both heat transfer and friction drag. This resulted in an ultimate regime in which the Stanton number and skin-friction coefficient became independent of the Reynolds number. However, the permeable wall was modelled using boundary conditions where the wall-normal velocity was proportional to the local pressure fluctuation. Kuwata, Tsuda & Suga (2020), using the lattice Boltzmann method, conducted DNSs of conjugate heat transfer in a turbulent duct flow that was partially filled with a porous medium consisting of square bars. They found the dispersive and turbulent heat fluxes inside the porous wall to be significant. However, they considered only one porous-wall configuration, consequently, it is not well understood how the heat transfer changes with permeability. The purpose of this study is therefore to investigate porous-wall turbulent heat transfer using DNSs that also take into account the heat transfer in the solid phase. The objective is to analyse the effect of different porous walls on heat transfer and flow turbulence. A brief overview of porous-wall turbulence is provided next.

1.2. Porous-wall turbulence

In their investigation of porous-wall turbulence, Gómez-de Segura & García-Mayoral (2019) used analytical solutions derived from Brinkman's equation (Brinkman 1949), $\nabla p = -\nu \mathbf{K}^{-1} \mathbf{u} + \tilde{\nu} \nabla^2 \mathbf{u}$, where $\mathbf{u}$ and p are the volume averaged velocity and pressure, respectively, and ν and $\tilde{\nu}$ are the molecular and the effective macroscopic viscosity, respectively, as boundary conditions to introduce the effect of anisotropic porous walls on a turbulent channel flow. Assuming a permeability tensor, $\mathbf{K}$, with only the principal components, K_x , K_y and K_z being non-zero (zero off-diagonal components), and streamwise-preferential configurations ($K_x > K_y$), they showed that turbulence remains smooth-wall-like for small values of the viscous-scaled effective wall-normal permeability ($\sqrt{K_y^+} < 0.6$). Beyond this regime, instabilities are triggered, giving rise to spanwise rollers and the flow transitioning to the so-called Kelvin–Helmholtz-like regime, hereafter called the K–H-like regime. The existence of these spanwise rollers had earlier been shown by Jiménez *et al.* (2001). Using linear stability analysis, they attributed the rollers to shear waves and K–H instabilities occurring at small/moderate to high permeability, respectively. This distinction between high- and low-permeability flow characteristics was also made by Manes, Poggi & Ridolfi (2011), who experimentally studied turbulent boundary layers over isotropic porous foams. They observed that only for the foam with the largest permeability ($\sqrt{K^+}=17.2$) did the flow exhibit the frequency associated with the K–H instability. Spanwise rollers have also been observed in riblet flows. Riblet-resolving linear stability analysis indicates that these rollers are not the most amplified K–H modes, but rather marginal modes with a streamwise wavelength of $\lambda_x^+ \approx 150$ in wall units (Camobreco *et al.* 2025). Similar wavelengths, significantly longer than the K–H instability wavelength, are observed in some flows over porous walls, suggesting that these rollers are likewise marginal modes. Although the spanwise rollers observed over low to intermediately permeable walls are not caused by a K–H instability, this flow regime is commonly called the K–H-like regime since the rollers are similar.

In a preceding study, Habibi Khorasani *et al.* (2024) conducted DNSs of turbulent open-channel flows over porous lattices with low to intermediate permeabilities. The low-permeability cases retained the near-wall flow characteristics of smooth-wall turbulence while the intermediate-permeability cases fell into the K–H-like regime. The approximate permeability threshold across which this regime change occurred was also in reasonable agreement with the linear stability predictions of Sharma, Gomez-de Segura &

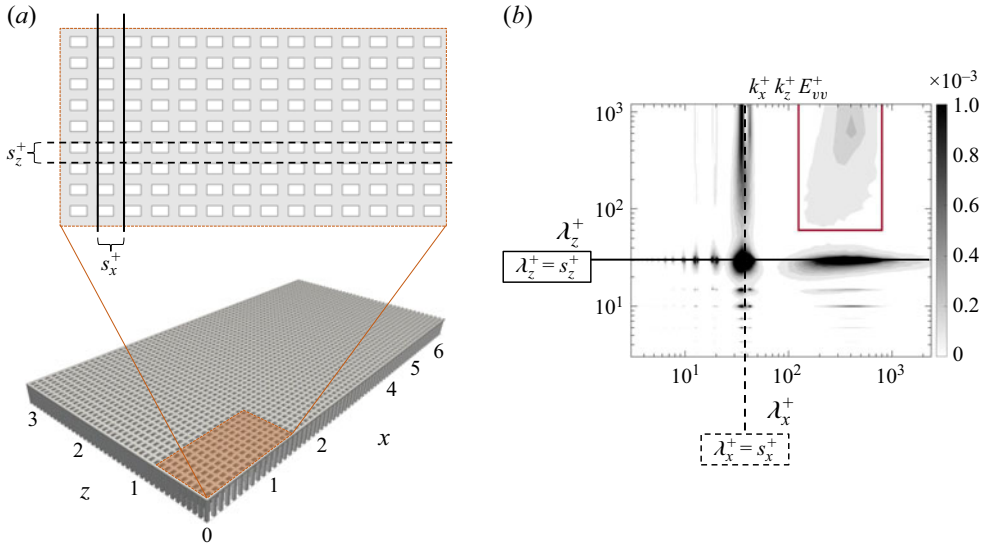


Figure 1. Illustration of how the texture-coherent flow scales are related to the physical pore dimensions: the region of the porous wall magnified in (a) has its streamwise and spanwise pitches, s_x^+ and s_z^+ , marked using the — and - - lines, respectively. The wall-normal pre-multiplied spectral energy $k_x^+ k_z^+ E_{vv}^+$ at the surface of the porous wall in case *KP1* (parameters are given in §2) is shown in (b), where the area enclosed by the dark-red line represents the ambient turbulence scales. The contours indicated by the — and - - lines in (b) are the principal energetic scales of the texture-coherent flow component that occur at streamwise and spanwise wavelengths λ_x^+ and λ_z^+ equal to the porous-wall pitches. The similar-looking but less intense contours at lower wavelengths are the harmonics of the texture-coherent flow.

García-Mayoral (2017), Gómez-de Segura *et al.* (2018) using their Brinkman-derived permeable-wall boundary conditions.

Aside from the permeability-triggered flow regime change, there is also the flow component induced by the presence of the porous wall. Any solid structure introduces spatial inhomogeneities into the flow field since the flow is forced to navigate around the solid obstructions. The resulting flow component has scales which conform to the spatial wavelengths of the porous wall. This flow component is commonly referred to as the texture-coherent or grain-coherent flow (Fairhall, Abderrahaman-Elena & García-Mayoral 2019; Hao & García-Mayoral 2025). Its presence is demonstrated in figure 1, which shows the pre-multiplied spectral energy of the wall-normal velocity, $k_x^+ k_z^+ E_{vv}^+$, (figure 1b) for the turbulent flow over the depicted porous wall (figure 1a). The energy spectrum reveals that, in addition to the turbulent scales of the flow (the area enclosed by the red lines), energetic scales exist at wavelengths equal to the wall-parallel pore spacings s_x^+ and s_z^+ , which are the signature of the texture-coherent flow. The texture-coherent flow undergoes modulation by the surrounding turbulence (Abderrahaman-Elena, Fairhall & García-Mayoral 2019) and the two can also directly interact with one another when their scales overlap. Such interactions make additional nonlinear contributions to the near-wall dynamics.

1.3. Purpose of this study

In this study, turbulent heat transfer over conductive porous lattices comprising cuboid pores covering a range of permeabilities, from small ($\sqrt{K^+} \approx 1$) to large ($\sqrt{K^+} \approx 19$), are numerically investigated. These porous walls span the range of smooth-wall-like to

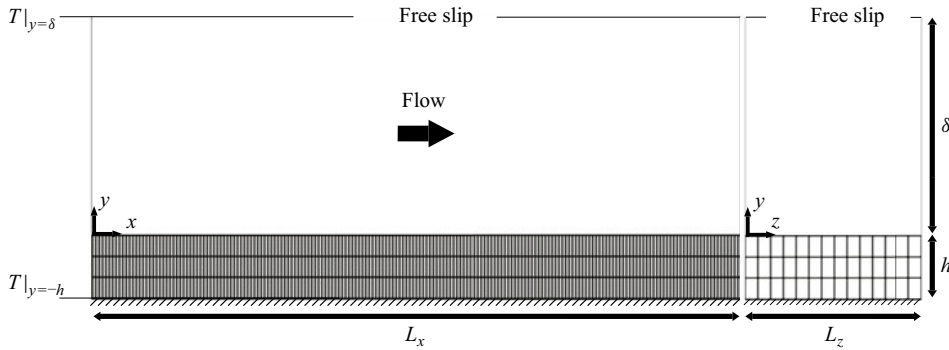


Figure 2. Sketch of the computational domain, similar to that of Habibi Khorasani *et al.* (2024).

K–H-like flow regimes, and also include configurations with scale overlap between the ambient turbulence and texture-coherent flow. This way, we can examine how heat transfer evolves over porous walls as we increase the permeability and pass through different regimes of porous-wall turbulence. It also allows for comparisons with heat transfer over rough walls in the transitional and fully rough regimes. Additionally, inclusion of conjugate heat transfer permits us to examine whether increasing permeability has a beneficial or detrimental effect on heat transfer. Heat-transfer efficiency is assessed using the Reynolds analogy, following Bunker (2013) and Rouhi *et al.* (2022). The dissimilarity in momentum and heat transfer (Reynolds analogy breakdown) over the porous walls is further studied in terms of near-wall flow events that contribute to the turbulent transport of heat and momentum.

2. Direct numerical simulation set-up

The numerical set-up used here is similar to that of Habibi Khorasani *et al.* (2024) and the interested reader can refer to that work for details. Only the most pertinent aspects as well as any differences are described in this section.

Direct numerical simulations of open-channel flow were conducted in the numerical domain depicted in figure 2. The dimensions of the domain are $(L_x, L_y, L_z) = (2\pi\delta, \delta + h, \pi\delta)$, where x, y, z refer to the streamwise, wall-normal and spanwise directions, δ is the channel height and $h = 0.3\delta$ the porous-wall thickness. The flow is governed by the incompressible Navier–Stokes equations

$$\nabla \cdot \mathbf{u} = 0, \quad (2.1a)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \Pi \mathbf{e}_x, \quad (2.1b)$$

where t is time, $\mathbf{u}(\mathbf{x}, t)$ is the velocity vector with (u, v, w) the streamwise, wall-normal and spanwise components, respectively, $p(\mathbf{x}, t)$ is the pressure, ρ is the density, ν is the kinematic viscosity, $\mathbf{e}_x$ is a unit vector in the streamwise direction and Π is the spatially uniform but temporally varying forcing term, adjusted such that a constant flow rate is maintained in the channel region. The domain is periodic along the x and z directions. No-slip and impermeability conditions are imposed at $y = -h$ while at $y = \delta$ free-slip and impermeability conditions are imposed. The surface of the porous wall is at $y = 0$. Heat transfer is incorporated by solving a convection–diffusion equation for a scalar temperature field

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla)T = \alpha \nabla^2 T, \quad (2.2)$$

where T , $\alpha = \kappa/(\rho c_p)$, c_p and κ are the temperature, thermal diffusivity, specific heat capacity and thermal conductivity, respectively. Fixed-temperature conditions $T|_{y=-h}$ and $T|_{y=\delta}$ are imposed at the bottom and top domain boundaries, respectively, so that heat is transferred from the bottom of the porous wall to the top of the channel region. The temperature is a passive scalar since buoyancy effects are neglected in the present study.

Equations (2.1)–(2.2) are solved using the open-source solver CaNS (Costa 2018; Costa *et al.* 2021; Diez Sanhueza *et al.* 2025), which discretises them using second-order central finite differences on structured Cartesian grids using a staggered arrangement. The grids used are uniform along x and z . In the y -direction, the grid is stretched in the channel region of the domain using a hyperbolic tangent function and uniform across the porous region.

The porous substrate has essentially the same structure as in the DNSs by Habibi Khorasani *et al.* (2024) and consists of a matrix of interconnected square bars with a cross-section of $d \times d$ aligned with the x , y and z directions. The bar spacings (itches) s_x , s_y and s_z in the streamwise, wall-normal and spanwise directions, respectively (see figure 1), are systematically varied, while $d = 0.0398$ is kept constant in the DNSs, resulting in a range of porosities and anisotropic permeabilities. The purpose of these variations is to obtain a meaningful flow transition with increasing permeability; from smooth-wall-like behaviour to flow states featuring spanwise rollers and strongly enhanced turbulence.

Similar porous structures composed of interconnected square bars have been used in numerical (Hao & García-Mayoral 2025) and experimental studies (Suga, Okazaki & Kuwata 2020; Vijay & Luhar 2024), as they are straightforward to characterise and relatively easy to fabricate (e.g. via additive manufacturing). The porous interface in the present study is chosen to be flat to isolate the effect of the permeability on the flow and heat transfer. Protruding elements at the interface can act as roughness and increase drag (Suga *et al.* 2020; Hao & García-Mayoral 2025). Previous work indicates that when the porous structure (i.e. bar spacing) is sufficiently fine ($\lesssim 50$ viscous wall units) the macroscopic properties, i.e. permeability, have a much larger impact on the mean flow, Reynolds stresses and spectra than the detailed structure of the porous substrate (Hao & García-Mayoral 2025). This observation suggests that the results of our study are also relevant for turbulent flows and heat transfer over other porous substrates with different structures but similar permeabilities. Only in the cases with the largest bar spacings can the texture-coherent flow directly interact with the overlying turbulence. In such cases, the specific structure of the porous substrate has a larger effect on the flow statistics and heat transfer.

The porous walls are numerically represented using the immersed-boundary method of Breugem & Boersma (2005) and Paravento, Pourquie & Boersma (2008) for grid-conforming Cartesian geometries. It involves modifying the discretised advective and diffusive fluxes of cells identified as solids such that the no-slip condition becomes exactly imposed, giving second-order spatial accuracy. The grid resolution used is the same as that in Habibi Khorasani *et al.* (2024), with the number of grid points being $(N_x, N_y, N_z) = (1620, 324, 810)$. The square bars are resolved with 10 grid points in the streamwise and spanwise directions, and 25 grid points in the wall-normal direction.

CaNS integrates the equations in time using a fractional-step pressure-correction algorithm. To remove the diffusive constraint on the time step used for temporal integration, the diffusive terms can be treated implicitly by solving a Helmholtz equation for each of the velocity components. However, to benefit from this in our conjugate

heat-transfer simulations that have a spatially varying thermal diffusivity in the porous wall, we use the method of Dodd & Ferrante (2014) to split the diffusive term into a variable term and a constant term, with the former treated implicitly and the latter explicitly. This prevents the greater thermal diffusivity in the solid phase from imposing a strict restriction on the time step.

The relevant non-dimensional quantities for characterising the flow are the bulk Reynolds number, $Re_b = U_b \delta / \nu$, and the Prandtl number, $Pr = \nu / \alpha$. The simulations were conducted at a targeted $Re_\tau = u_\tau \delta / \nu = 360$, defined based on the channel region of the domain ($0 \leq y \leq \delta$) and achieved by adjusting Re_b . Here, $u_\tau = \sqrt{\tau_w / \rho}$ is the friction velocity and τ_w is the mean shear stress at $y = 0$. At this Re_τ , $h^+ \approx 100$ and therefore the porous substrate can be considered deep and no substantial change in the overlying turbulent flow is expected when h is further increased (Hao & García-Mayoral 2025). For the thermal property of the working fluid, $Pr = 0.71$ was used, which corresponds to air. A thermal diffusivity of $\alpha_s = 4.4\alpha_f$ was set for the solid phase to make it analogous to aluminium.

Direct numerical simulations of turbulent channel flow over porous walls by Hao & García-Mayoral (2025) showed that the mean velocity, near-wall fluctuation profiles and spectra agree well at $Re_\tau = 180, 360$ and 550 when the porous substrate dimensions (and thus the permeability) are matched in wall units. Hao & García-Mayoral (2025) also considered four porous-wall cases with a flat interface, as in the present study and Habibi Khorasani *et al.* (2024), and found that the mean-velocity profile shift in their DNS at $Re_\tau = 180$ and ours at $Re_\tau = 360$ follows the same trend with permeability. These observations suggest that the findings in the present study extend to other Reynolds numbers.

Variables ϕ are decomposed using the Reynolds decomposition

$$\phi(x, y, z, t) = \Phi(y) + \phi'(x, y, z, t), \quad (2.3)$$

where ϕ is the instantaneous value, Φ is the time- and plane-averaged mean value and ϕ' is the full time-and-space varying fluctuation. The latter can be further decomposed into (Reynolds & Hussain 1972)

$$\phi'(x, y, z, t) = \tilde{\phi}(x, y, z) + \phi''(x, y, z, t), \quad (2.4)$$

where $\tilde{\phi}(x, y, z)$ is the time-invariant but spatially varying dispersive component induced by the porous wall and $\phi''(x, y, z, t)$ the remaining time-and-space varying fluctuation due to turbulence.

Plane-averaged flow and temperature statistics are reported as fluid-averaged and solid-averaged quantities over a region S_i

$$\Phi_i(y) = \frac{1}{A_i} \iint_{S_i} \hat{\phi}(x, y, z) \, dx \, dz, \quad (2.5)$$

where A_i is the area of that region, $\hat{\phi}(x, y, z)$ is the time-averaged value of $\phi(x, y, z, t)$ and $i = f, s$ indicates over which phase the plane averaging has been done (f for fluid and s for solid). Since the flow velocities are zero in the solid-occupied regions, all velocity-related statistics are fluid-phase averages, and so the subscript f is dropped. The temperature field, however, is present throughout both phases. The '+' superscript indicates viscous-scaled quantities using u_τ and ν . The non-dimensional temperature θ is defined as

Case	Symbol	Re_τ	φ	s_x^+	s_y^+	s_z^+	$\sqrt{K_x^+}$	$\sqrt{K_y^+}$	$\sqrt{K_z^+}$	$\sqrt{\frac{K_x^+}{K_y^+}}$
<i>KP1</i>	●	364	0.75	36.0	108.0	28.0	3.68	3.96	5.45	0.93
<i>KP1a</i>	■	361	0.86	56.0	108.0	42.0	7.96	8.20	10.09	0.97
<i>KP1b</i>	★	364	0.95	108.0	108.0	84.0	18.97	18.93	19.84	1.00
<i>KP2</i>	●	361	0.68	36.0	54.0	28.0	3.31	3.17	4.64	1.04
<i>KP2a</i>	■	362	0.81	56.0	54.0	42.0	6.43	6.46	7.43	1.00
<i>KP2b</i>	★	363	0.92	108.0	54.0	84.0	12.66	15.32	12.46	0.83
<i>KP3</i>	●	364	0.61	36.0	36.0	28.0	2.79	2.79	3.57	1.00
<i>KP3a</i>	■	365	0.76	56.0	36.0	42.0	4.80	5.77	5.09	0.83
<i>KP3b</i>	★	362	0.86	108.0	36.0	84.0	7.71	12.69	6.99	0.61
<i>TP</i>	●	357	0.50	28.0	27.0	28.0	2.11	2.16	2.11	0.98
<i>SP</i>	●	359	0.43	21.0	54.0	21.0	1.59	1.11	1.59	1.43
<i>ZP</i>	●	363	0.0	0.0	0.0	0.0	0.0	0.0	0.0	—

Table 1. Parameters of the DNS cases. The pore pitch lengths are s_x^+ , s_y^+ and s_z^+ , while $\sqrt{K_x^+}$, $\sqrt{K_y^+}$ and $\sqrt{K_z^+}$ are the effective permeabilities which are equivalent to the permeability Reynolds number, Re_K , used throughout the literature. The ratio of streamwise to wall-normal permeability serves as the measure of a wall's anisotropy.

$$\theta = \frac{T - T|_{y=\delta}}{T|_{y=-h} - T|_{y=\delta}}, \quad (2.6)$$

so that $\theta = 1$ at $y = -h$ and $\theta = 0$ at $y = \delta$.

A summary of all DNS cases and their relevant parameters is given in [table 1](#). Here, the porosity φ is the volume fraction of the fluid in a representative elementary volume (Habibi Khorasani *et al.* 2024). The zero-permeability case, *ZP*, is the baseline smooth-wall case where the wall region is a solid impenetrable block. The smooth-wall-like permeable case, *SP*, has low permeability and retains the smooth-wall-like structure of near-wall turbulence. The transitionally permeable case, *TP*, shows signs of departing the smooth-wall-like regime. The K–H permeable cases, *KP*($\langle$), fall into the K–H-like regime of wall turbulence. We use here and hereafter the notation *KP*($\langle$) to refer collectively to the subcases *KP1*, *KP2* and *KP3*. The walls of cases *SP*, *TP*, *KP3*, *KP2* and *KP1*, respectively, correspond to cases *LP1*, *MP*, *HP3*, *HP2* and *HP1* of Habibi Khorasani *et al.* (2024), where it was established to which flow regime each case belonged. Habibi Khorasani *et al.* (2024) also provide schematics and visualisations of some of the considered porous substrates. The off-diagonal permeabilities of the porous structures are zero, while the diagonal permeabilities, $\sqrt{K_i^+}$, were obtained using Darcy's law, $\nabla p = -\nu K^{-1} \cdot \mathbf{u}$, by conducting Stokes simulations in representative elementary volumes of each porous wall. The *KP*($\langle$)*a* and *KP*($\langle$)*b* cases retain the same wall-normal pitches, s_y , as the *KP*($\langle$) case but have successively larger wall-parallel pitches, s_x and s_z . The purpose of this successive increase in s_x and s_z , in addition to increasing permeability, was to cause the texture-coherent scales to overlap with those of the ambient turbulence as discussed in § 1.2, and to further enhance the heat transfer.

In all cases, $d^+ = 14$ and the spanwise and streamwise grid spacing is 1.4 wall units, which is sufficient to resolve the turbulence. The wall-normal grid spacing is uniform and 0.56 wall units in the porous substrate, whereas it is stretched away from the porous wall. Further details and a resolution assessment for the flow in and over the porous substrate

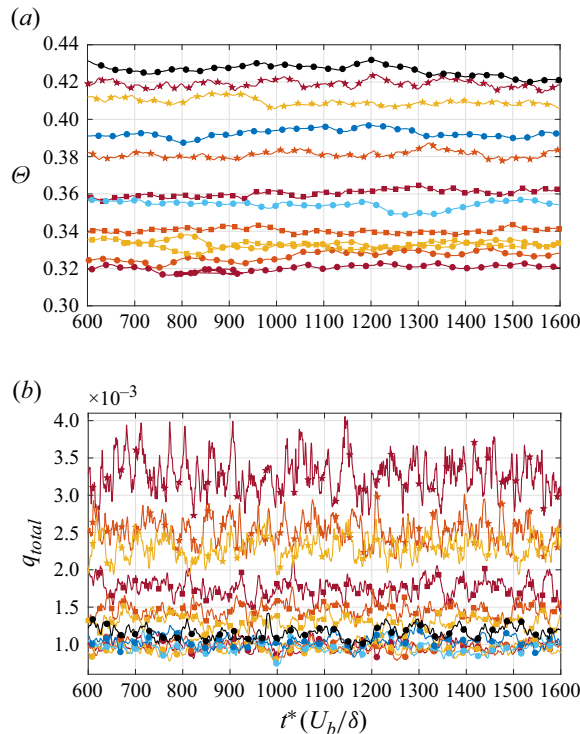


Figure 3. Time history of (a) the bulk temperature and (b) the total heat flux at the top boundary.

are provided in Habibi Khorasani *et al.* (2024), noting that the present study uses the same resolution. The $KP\langle \rangle$ cases use 15 – 67 and 10 – 50 grid points per gap in the streamwise and spanwise directions, respectively. The grid sensitivity study by Habibi Khorasani *et al.* (2024) shows that 15 and 10 grid points per gap in the streamwise and spanwise directions, respectively, is sufficient to resolve the flow in and over the porous substrate. Kuwata *et al.* (2020) performed DNSs of conjugate heat transfer in duct flow with a porous wall composed of square bars using the same Pr and solid to fluid thermal diffusivity ratio as used here. They compared a simulation using a 23×23 grid to resolve each bar with a second case with twice the grid spacing and found no significant differences. In our DNSs, the streamwise and spanwise resolutions per bar are comparable to their coarse case, whereas the wall-normal resolution is comparable to their refined case. Our resolution should therefore be adequate to resolve both the flow and heat transfer.

Each DNS case of table 1 was first run for a time period of $2000\delta/U_b$ so that the flow and temperature fields reached fully developed, statistically stationary conditions. The DNSs were then continued for an additional time period of $2000\delta/U_b$ to collect statistics. Statistical steadiness is evidenced in figure 3, which shows a part of the time history of the bulk temperature and total heat flux.

3. Results

3.1. Flow differences over the different porous walls

Before analysing the heat transfer, the velocity fields for the cases of table 1 in the channel region ($y > 0$) are briefly examined to establish their features. Since heat transfer

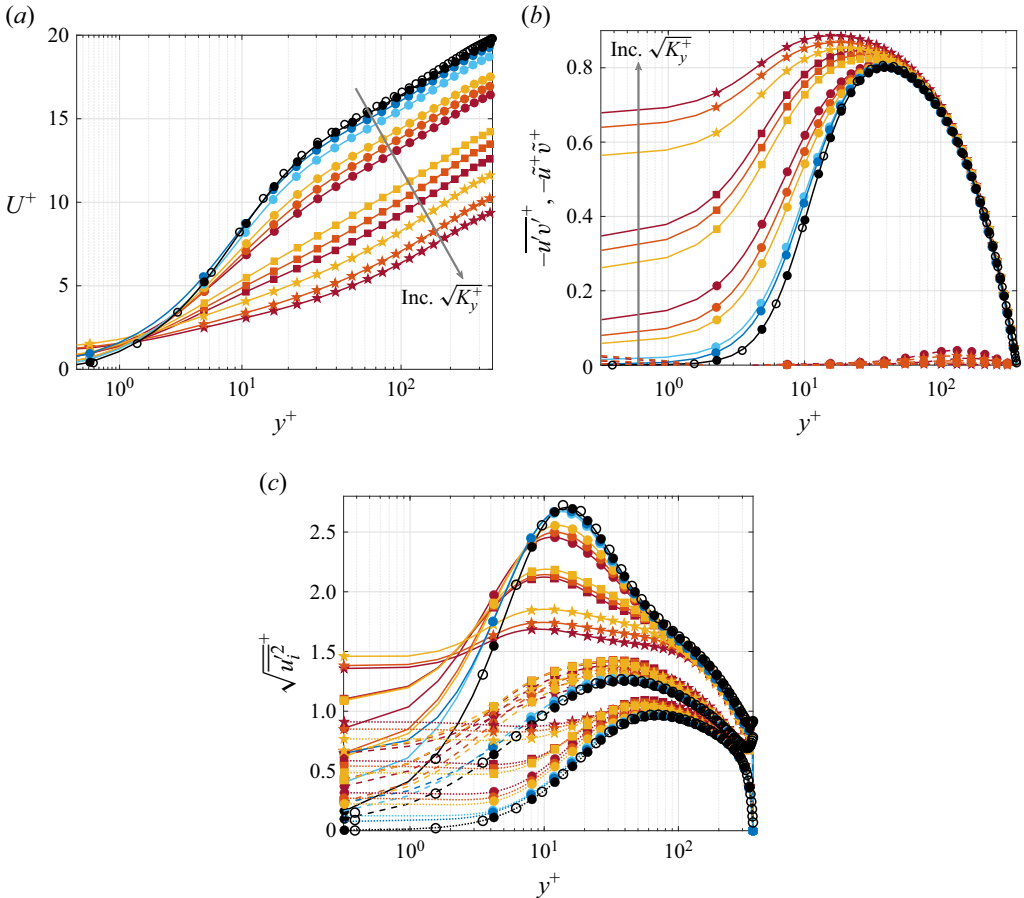


Figure 4. Profiles of the (a) mean velocity, (b) Reynolds shear stress, (c) root-mean-square velocity fluctuations. The direction of the grey arrow in (a) and (b) indicates increasing wall-normal permeability. In (b); —, total Reynolds stress; — —, dispersive Reynolds stress. In (c); —, total streamwise fluctuations; — —, total spanwise fluctuations; ···, total wall-normal fluctuations. The open circles, $\circ$, are reference smooth-wall data produced using a pseudo-spectral solver.

is passive, structural changes in the velocity field will largely determine any differences that emerge in the temperature fields for the different walls.

Figure 4(a) shows the plane- and time-averaged mean-velocity profiles. It is clear that, as the permeability grows, the mean-velocity deficit relative to the smooth-wall case, ZP , also becomes larger. The mean-velocity profile in the low-permeability case SP does not differ much from that in case ZP , but in the high-permeability cases, and especially in $KP(\rangle)b$, the difference is large. The differences observed in figure 4(a) for the mean velocities are reflected in figure 4(b) for the total Reynolds shear stress profiles, $-\overline{u'v'}^+$. Here and below, an overline denotes time and planar averaging. Obviously, the $KP(\rangle)$, $KP(\rangle)a$ and $KP(\rangle)b$ profiles fall into separate groups in the near-wall region. The strength of the Reynolds shear stress progressively grows in this region as the wall becomes more permeable and is more uniform. These higher stress levels at the surface cause the lower mean velocities shown in figure 4(a). The change of $-\overline{u'v'}^+$ profiles indicates structural changes in turbulence. This change can also be inferred from the root-mean-square (r.m.s.) velocity fluctuations in figure 4(c). The near-wall peak of the u velocity fluctuations strongly decreases as the wall

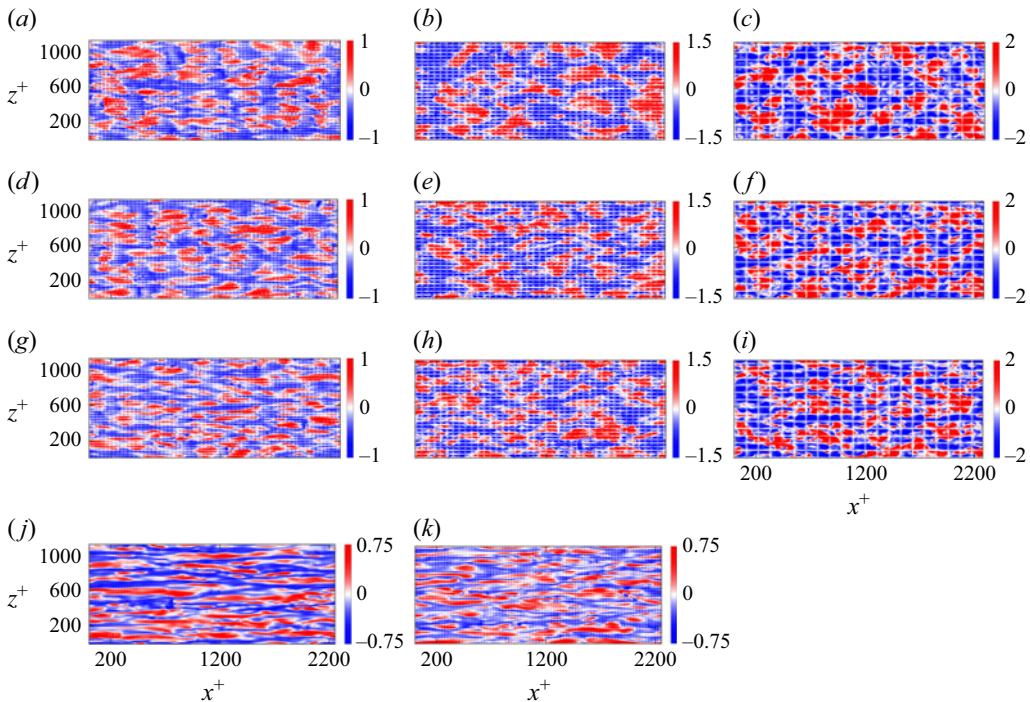


Figure 5. Instantaneous snapshots of streamwise velocity fluctuations at $y = 0$ normalised by u_τ ; (a) KP1, (b) KP1a, (c) KP1b, (d) KP2, (e) KP2a, (f) KP2b, (g) KP3, (h) KP3a, (i) KP3b, (j) SP, (k) TP.

becomes more permeable. The fluctuations of w and v , however, grow more intense, such that the turbulence becomes more isotropic. A higher permeability also allows turbulent motions to penetrate into the wall, leading to a rise in the r.m.s. values for all three velocity components. While not shown here, it can readily be deduced from figures 4(b) and 4(c) that, in the K–H-like KP cases, the turbulent stresses penetrate into the wall. This penetration is quite deep for the highly permeable KP(b) cases, as will become clear later. The mean and fluctuating velocity profiles of case ZP in figures 4(a), 4(b) and 4(c) agree well with reference smooth-wall open-channel DNS data at $Re_\tau = 360$ obtained with a pseudospectral solver (Chevalier *et al.* 2014), as used in e.g. Brethouwer (2017).

Figure 4(b) also shows the planar-averaged dispersive component of the Reynolds shear stress for the highly permeable cases KP1, KP1a, KP1b, KP2, KP2a and KP2b. It is evident that, even for the most permeable walls, the dispersive contribution to the total Reynolds shear stress throughout the bulk flow region is negligible. The dispersive shear stress does appear to grow around $y^+ \approx 150$ with growing permeability (and hence stronger K–H-like rollers), and is slightly negative close to the porous wall. This negative shear stress is caused by the K–H-like rollers, which can induce negative shear (Endrikat *et al.* 2021; Rouhi *et al.* 2022). Also notable is that, unlike in the case of the canopy structures studied by Chen & García-Mayoral (2023), no strong dispersive stresses occur at the surface of the porous wall.

The changes in flow turbulence structure arising in the different cases can be seen in the instantaneous flow-field snapshots of figures 5 and 6. Figure 5 depicts streamwise velocity fluctuations at the surface ($y = 0$). The low-permeability case SP remains smooth-wall-like, with streamwise elongated streaky patches (figure 5j). As the porous wall becomes more permeable, the streaky structure becomes disrupted and groupings of smaller patches

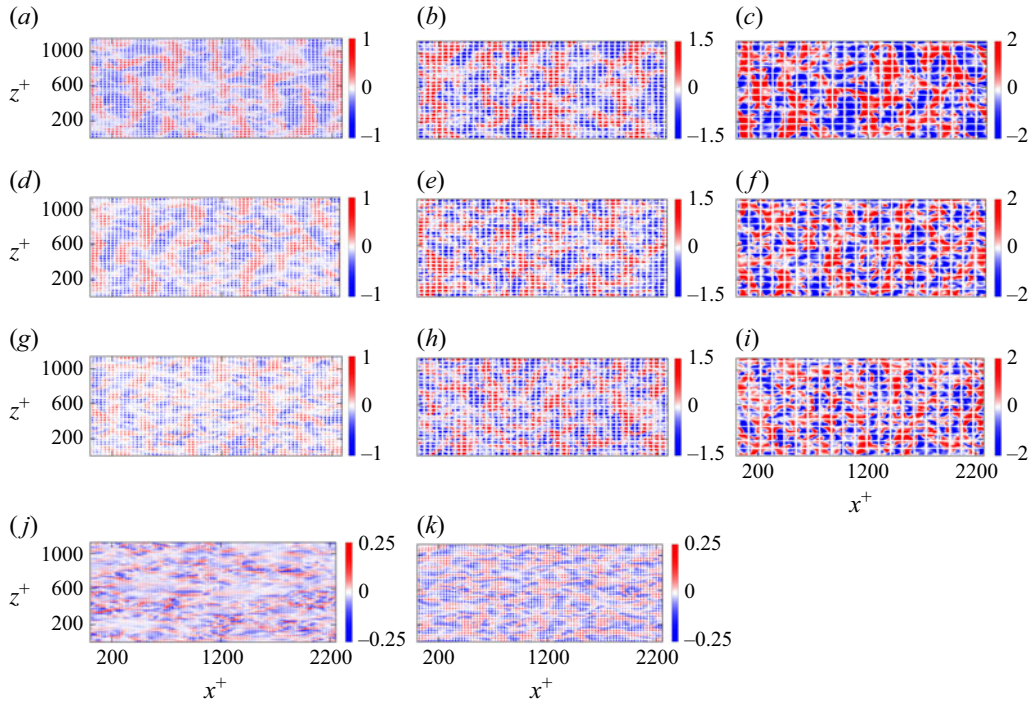


Figure 6. Instantaneous snapshots of wall-normal velocity fluctuations at $y = 0$ normalised by u_τ ; (a) KP1, (b) KP1a, (c) KP1b, (d) KP2, (e) KP2a, (f) KP2b, (g) KP3, (h) KP3a, (i) KP3b, (j) SP, (k) TP.

appear, such as those in figures 5(d) and 5(a). These patches are spanwise aligned and get convected downstream. The change is even more notable when examining the wall-normal velocity in figures 6(d) and 6(a), where lengthy spanwise-oriented patches are seen. The larger horizontal pore dimensions in $KP\langle\rangle b$ by almost a factor of three leads to approximately a quadrupling of the effective wall-normal permeability, $\sqrt{K_y^+}$. As a result, more disorganised patches appear at the surface than in the $KP\langle\rangle$ cases, see figures 5(c,f) and 6(c,f).

As a final assessment of the flow over the different walls, and to relate the present observations to the description of porous-wall turbulent flows given in § 1.2, the spectral energies of the velocity fluctuations at the surface ($y = 0$) are examined in figure 7. The smooth-wall-like SP case clearly shows a different spectral signature for u , v energy spectra and the uv co-spectrum in figures 7(j), 7(k) and 7(l) than those of $KP1$ in figures 7(g), 7(h) and 7(i). The latter exhibit energetic large spanwise length scales which are absent in SP . These scales belong to the K–H-like structures that were described in § 1.2. Previously (Habibi Khorasani *et al.* 2024), it was shown that the spanwise structures observed in case $KP1$ (corresponding to case $HP1$ in Habibi Khorasani *et al.* 2024) are not due to a K–H instability since the frequency of these K–H-like structures was much higher than that of a K–H instability. However, since these structures share similarities with the K–H rollers we call them K–H-like structures. The signature of texture-coherent flow as described in figure 1 is visible for both SP and $KP1$, but is more energetic in the latter case. By increasing the streamwise and spanwise pitches (s_x^+ , s_z^+) of the pores in case $KP1a$, the related scales move closer to the ambient turbulent scales, leading to direct interactions. Further increasing the pitches in case $KP1b$ results in significant

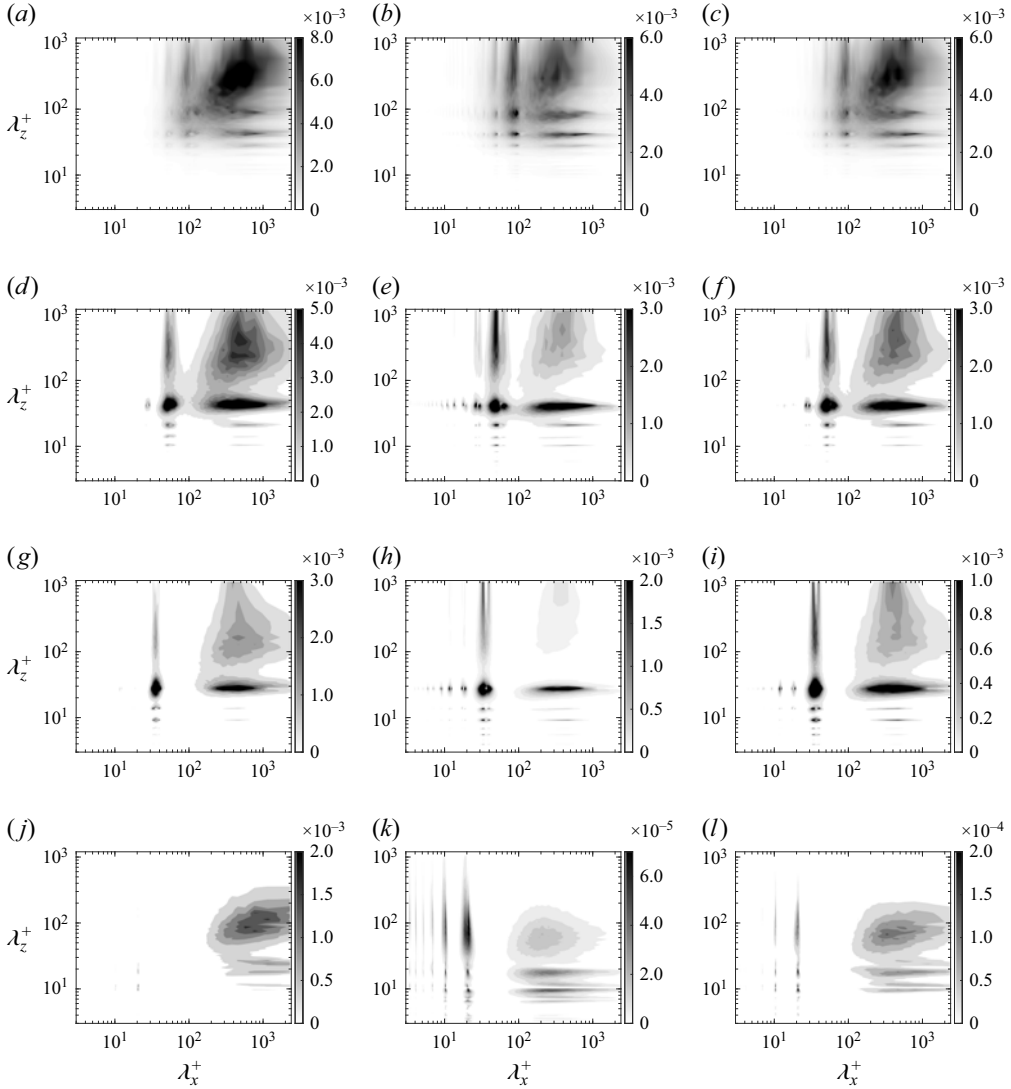


Figure 7. Premultiplied spectral energies $k_x^+ k_z^+ E_{uu}^+$, $k_x^+ k_z^+ E_{vv}^+$ and $k_x^+ k_z^+ E_{uv}^+$ at $y=0$: (a–c) *KP1b*, (d–f) *KP1a*, (g–i) *KP1*, (j–l) *SP*; (a) *KP1b*, $k_x^+ k_z^+ E_{uu}^+$, (b) *KP1b*, $k_x^+ k_z^+ E_{vv}^+$, (c) *KP1b*, $k_x^+ k_z^+ E_{uv}^+$, (d) *KP1a*, $k_x^+ k_z^+ E_{uu}^+$, (e) *KP1a*, $k_x^+ k_z^+ E_{vv}^+$, (f) *KP1a*, $k_x^+ k_z^+ E_{uv}^+$, (g) *KP1*, $k_x^+ k_z^+ E_{uu}^+$, (h) *KP1*, $k_x^+ k_z^+ E_{vv}^+$, (i) *KP1*, $k_x^+ k_z^+ E_{uv}^+$, (j) *SP*, $k_x^+ k_z^+ E_{uu}^+$, (k) *SP*, $k_x^+ k_z^+ E_{vv}^+$, (l) *SP*, $k_x^+ k_z^+ E_{uv}^+$.

scale overlap, which consequently causes the texture-coherent and ambient turbulence scales to be less distinguishable. The energy is more evenly distributed across a wide range of scales. Cases *KP2* and *KP3* follow the same trend as *KP1* with regards to the change in flow structure with increasing (s_x^+, s_z^+) and are not shown here for the sake of brevity.

The content of this section was merely to serve as a basis for the heat-transfer analysis and not as an exhaustive analysis of porous-wall turbulence. For more details regarding such flows, the references provided in § 1.2 can be consulted. The results are broadly consistent with those of Hao & García-Mayoral (2025), who observed smooth-wall-like

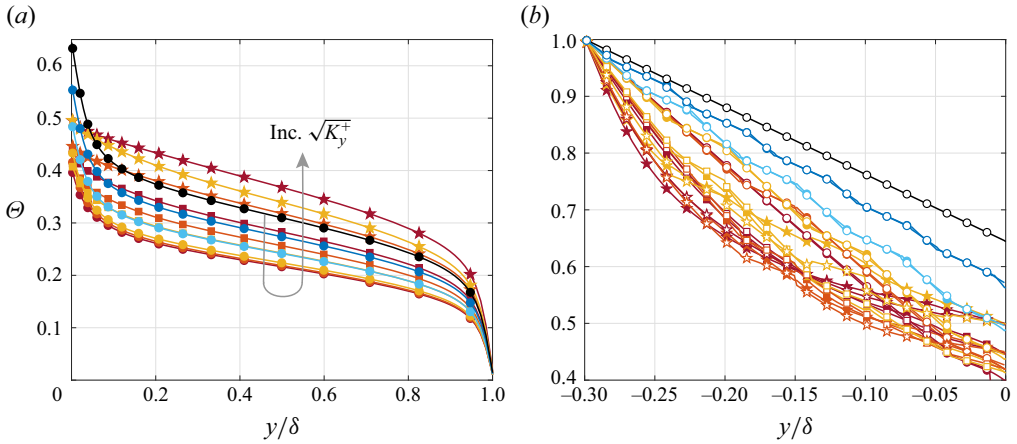


Figure 8. Mean temperature profiles in (a) channel region and (b) porous-wall region. Filled symbols are the fluid-phase-averaged temperature and open symbols are the solid-phase-averaged temperature. The direction of the arrow in (a) indicates increasing wall-normal permeability.

behaviour only for $K^+ \lesssim 1$. At higher permeabilities, significant changes were found in the Reynolds stresses and near-wall structures. They also found that permeability is the dominant factor influencing the overlying turbulence, while the granularity and structure of the porous substrate play lesser roles.

3.2. Temperature field statistics

Having established the main flow characteristics, the temperature field and heat transfer of the cases can now be examined. First, the profiles of the mean temperature Θ are examined in figure 8. Initially, as the wall becomes more porous, the temperature in the channel region (figure 8a) declines compared with case *ZP* with a solid wall. This decline means that the heat transport in the porous wall becomes less efficient relative to that in the channel region, which is likely due to the replacements of highly conductive solid material with lower conductive fluid with increasing permeability. This temperature decline persists in the $KP(\cdot)a$ cases, after which the temperature in the channel region begins to rise as the wall-normal permeability, $\sqrt{K_y^+}$, grows. This rise continues in the cases *KP1b*, *KP2b* and *KP3b*, where the mean temperature exceeds that of *ZP*. These are the cases with the highest wall-normal permeabilities ($\sqrt{K_y^+} > 10$) and overlapping texture-coherent and ambient turbulence scales, as was shown earlier in figure 7. Another change in the temperature profile is the decreasing mean gradient near the surface as $\sqrt{K_y^+}$ becomes larger, which leads to a more uniform profile. This can be attributed to the intensified turbulent mixing near the porous wall, as previously seen in figure 4(b) for the Reynolds shear stresses.

Figure 8(b) shows the mean temperature profile in the porous or solid wall using (2.5) to obtain the fluid-averaged and solid-averaged statistics. In case *ZP* the profile is linear since the heat transfer is purely conductive. As the wall becomes more permeable, it becomes cooler than in the *ZP* case, and the temperature becomes less linear. In the highly permeable-wall cases $KP(\cdot)b$, the temperature profiles become more convex with steeper gradients deeper in the porous wall and reduced gradients near the surface. The plane-averaged temperature is nearly equal in the solid and fluid phases, with only some differences noticeable in the $KP(\cdot)b$ cases.

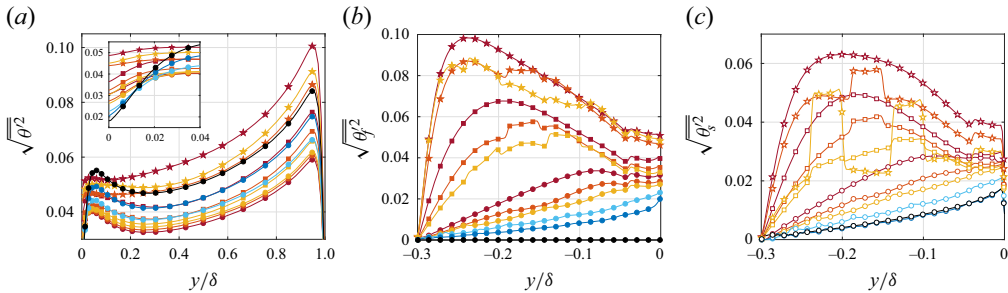


Figure 9. The r.m.s. temperature fluctuation profiles: (a) profiles in the channel region; (b) fluid-phase-averaged profiles in the porous region; (c) solid-phase-averaged profiles in the porous region.

Figure 9 shows the r.m.s. temperature fluctuations throughout the domain. In the channel region (figure 9a), the fluctuations decline when going from cases *ZP* to *KP3*, then *KP2* and finally *KP1*. The fluctuations then subsequently increase when going from *KP1* to *KP1b* with increasing $\sqrt{K_y^+}$ (*KP1* $\rightarrow$ *KP3a* $\rightarrow$ *KP2a* $\rightarrow$ *KP1a* $\rightarrow$ *KP3b* $\rightarrow$ *KP2b* $\rightarrow$ *KP1b*), with only the *KP(\cdot)b* cases having stronger fluctuations than *ZP*. The region immediately above the surface is an exception. There, the fluctuations consistently grow with permeability, as shown in the inset plot of figure 9(a). The near-surface peak also moves closer to the surface as the wall becomes more permeable. This is caused by the increased turbulent heat flux at the surface, as it leads to a greater production of temperature variance (Leonardi *et al.* 2015). Case *ZP* has the highest near-surface peak due to the large mean temperature gradient there.

Figures 9(b) and 9(c) show a greater distinction in the r.m.s. temperature fluctuations of the *KP(\cdot)a* and *KP(\cdot)b* cases from the others in the porous wall. Notably, the fluid-phase-averaged profiles in figure 9(b) show a steady increase in temperature fluctuations as the wall becomes more permeable (*ZP* $\rightarrow$ *KP1*, $\sqrt{K_y^+} = 0 \rightarrow \sqrt{K_y^+} \approx 4$), with a peak emerging at $y < -0.1$. The fluctuations decay toward the bottom of the wall. With increasing permeability (*KP3a* $\rightarrow$ *KP2a* $\rightarrow$ *KP1a*, $\sqrt{K_y^+} \approx 5.8 \rightarrow \sqrt{K_y^+} \approx 6.5 \rightarrow \sqrt{K_y^+} = 8.2$), the fluctuations increase notably and the peak moves toward the lower half of the wall. Consequently, the fluctuations become stronger further down into the wall. This trend persists for the highest-permeability cases (*KP3b* $\rightarrow$ *KP2b* $\rightarrow$ *KP1b*, $\sqrt{K_y^+} \approx 12.7 \rightarrow \sqrt{K_y^+} \approx 15.3 \rightarrow \sqrt{K_y^+} \approx 18.9$), with the fluctuations growing steadily towards the bottom wall, where they first attain a peak and the subsequently decay to zero due to the imposed temperature boundary condition. In the latter cases, the position of the peak is at $y/\delta \approx -0.25$ close to the bottom of the wall. In case *KP1b*, the peak in the wall is almost equal to the peak in the channel region.

The observations for the fluid-phase r.m.s. profiles in figure 9(b) also hold for the solid-phase profiles in figure 9(c), with the main difference being the overall smaller fluctuations than in the fluid phase. Notable jumps occur in the profile of *KP3b* as it passes through parts with changes in solid fraction and related conductive capacity. Such jumps are also visible in some of the other cases, specifically those with a higher permeability, but they are not as pronounced as in *KP3b*. This could be a consequence of the higher anisotropy of the porous substrate in *KP3b* than in the other cases.

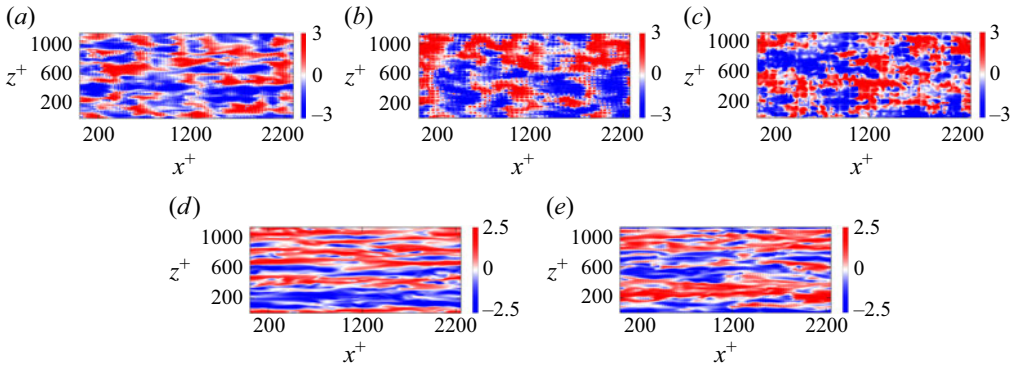


Figure 10. Instantaneous snapshots of temperature fluctuations at $y = 0$ for select cases of [table 1](#); (a) KP1, (b) KP1a, (c) KP1b, (d) SP, (e) TP.

The overall picture of the temperature fluctuations is reflective of the velocity fluctuations observed in [figure 4](#). Temperature fluctuations intensify near the surface as it becomes more permeable ([figure 9a](#)). This intensification is due to the enhanced mixing caused by the large-scale K–H-like rollers, which also produce high Reynolds shear stresses ([figure 4b](#)). The temperature field at $y = 0$, visualised in [figure 10](#), shows the same structure evolution as the streamwise velocity field visualised in [figure 5](#). The initial streak-like patterns in the temperature field become disrupted and break down into short patches as the wall becomes more permeable. However, in the highest-permeability case *KP1b* ([figure 11c](#)), the patches have higher streamwise coherence than those of the streamwise velocity ([figure 5c](#)). This streamwise coherence is also reflected in the spectra of the temperature fluctuations for cases *KP1*, *KP1a* and *KP1b* shown in [figure 11\(a,b,c\)](#), where energy remains concentrated at large streamwise wavelengths. Also the signature of the K–H-like structures at long spanwise wavelengths is present in both the spectra of temperature fluctuations and wall-normal turbulent heat flux shown in [figure 11\(d,e,f\)](#). The intense temperature fluctuations deep inside the porous wall in the cases with large permeabilities is indicative of strong turbulent heat fluxes, which will be examined next.

The total turbulent heat flux $\overline{\theta'v'}$, normalised by U_b , is shown in [figure 12\(a\)](#). It is clear that, as $\sqrt{K_y^+}$ becomes larger, the turbulent heat flux grows near the surface. The low-permeability cases *SP* and *TP* show only small gains in $\overline{\theta'v'}$ close to the surface. When the K–H-like instability sets in (beyond *TP*), $\overline{\theta'v'}$ grows almost consistently in the porous wall and channel region with increasing $\sqrt{K_y^+}$. Cases *KP3b*, *KP2b* and *KP1b* with their very high permeabilities ($\sqrt{K_y^+} \approx 12.7$, $\sqrt{K_y^+} \approx 15.3$, $\sqrt{K_y^+} \approx 18.9$) show large $\overline{\theta'v'}$ at the surface, explaining why the mean temperature profiles of these cases lacked a diffusive characteristic near the surface ([figure 8a](#)). The change in $\overline{\theta'v'}$ in the channel region ($y/\delta > 0$) mirrors that of the temperature fluctuations ([figure 9a](#)). The turbulent heat flux initially declines when going from cases *ZP* to *KP1* (*ZP* $\rightarrow$ *SP* $\rightarrow$ *TP* $\rightarrow$ *KP3* $\rightarrow$ *KP2* $\rightarrow$ *KP1*), but grows when going from cases *KP1* to *KP1b* (*KP1* $\rightarrow$ *KP3a* $\rightarrow$ *KP2a* $\rightarrow$ *KP1a* $\rightarrow$ *KP3b* $\rightarrow$ *KP2b* $\rightarrow$ *KP1b*). The *KP(a)* and *KP(b)* cases have larger $\overline{\theta'v'}$ than the baseline *ZP* case. Similar to the dispersive Reynolds shear stresses ([figure 4b](#)), the dispersive wall-normal heat flux in [figure 12\(b\)](#) is negligible. The heat transport is thus mainly caused by turbulence and K–H-like rollers.

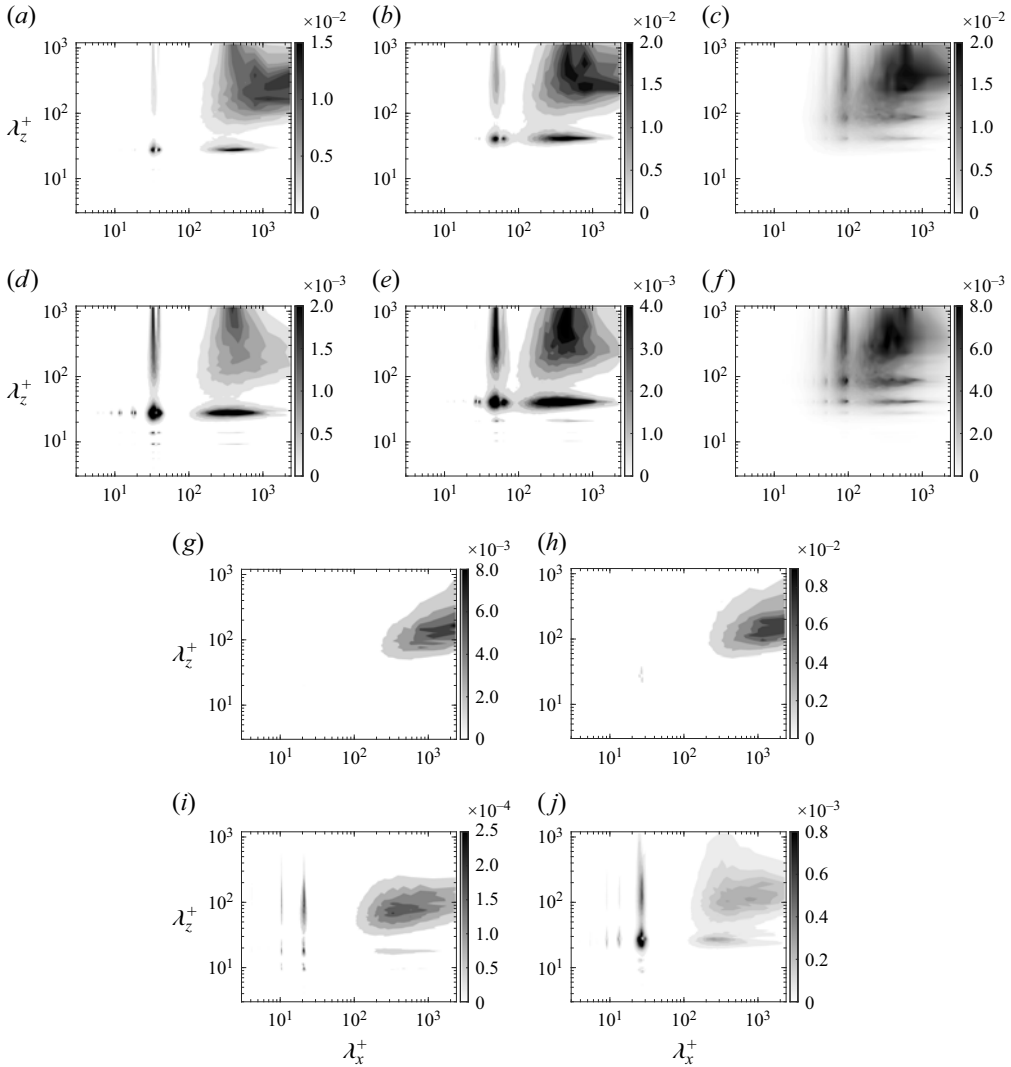


Figure 11. Premultiplied spectral energies $k_x^+ k_z^+ E_{\theta\theta}^+$ and $k_x^+ k_z^+ E_{\theta v}^+$ at $y=0$ for the cases shown in figure 10; (a) $KP1$, $k_x^+ k_z^+ E_{\theta\theta}^+$, (b) $KP1a$, $k_x^+ k_z^+ E_{\theta\theta}^+$, (c) $KP1b$, $k_x^+ k_z^+ E_{\theta\theta}^+$, (d) $KP1$, $k_x^+ k_z^+ E_{\theta v}^+$, (e) $KP1a$, $k_x^+ k_z^+ E_{\theta v}^+$, (f) $KP1b$, $k_x^+ k_z^+ E_{\theta v}^+$, (g) SP , $k_x^+ k_z^+ E_{\theta\theta}^+$, (h) TP , $k_x^+ k_z^+ E_{\theta\theta}^+$, (i) SP , $k_x^+ k_z^+ E_{\theta v}^+$, (j) TP , $k_x^+ k_z^+ E_{\theta v}^+$.

In the porous wall ($y/\delta < 0$), $\overline{\theta'v'}$ is negligible in cases SP and TP , while in cases $KP3$, $KP2$ and $KP1$, $\overline{\theta'v'}$ penetrates deeper in the porous wall, down to $y/\delta \approx -0.1$, before dissipating. Cases $KP\langle a \rangle$ and $KP\langle b \rangle$ have large $\overline{\theta'v'}$ in the shallow part of the wall ($-0.1 < y/\delta < 0$) and develop a peak before declining when approaching the bottom wall. The turbulent flux $\overline{\theta'v'}$ in these cases is much higher than in $KP\langle \rangle$ and penetrates deep into the porous wall. In all KP cases, the peak of $\overline{\theta'v'}$ in the porous wall approaches $\overline{\theta'v'}$ in the channel region. Only cases $KP\langle a \rangle$ and $KP\langle b \rangle$ show a notable dispersive heat flux in the porous wall (figure 12b).

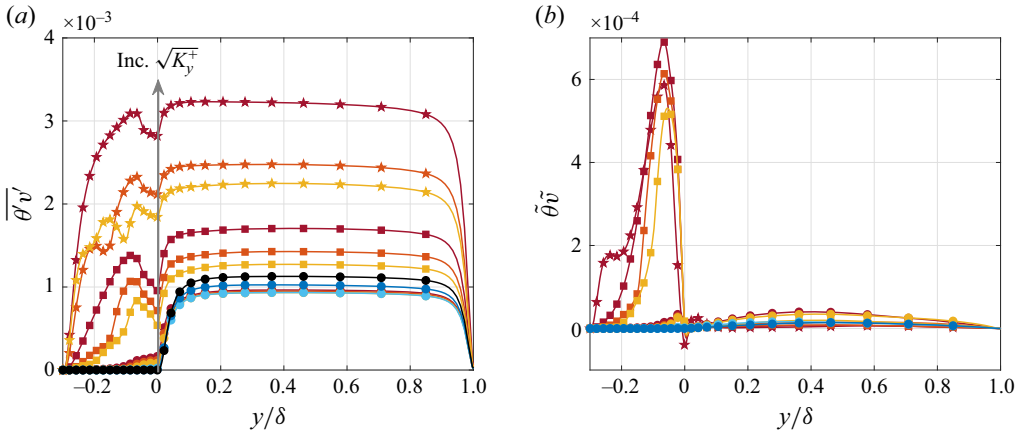


Figure 12. Wall-normal heat flux in the whole domain: (a) total wall-normal heat flux and (b) dispersive wall-normal heat flux. Direction of the grey arrow in (a) indicates increasing wall-normal permeability.

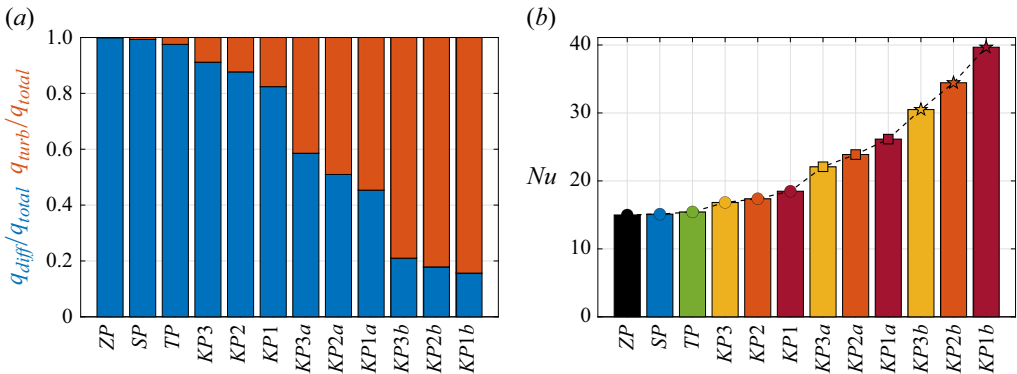


Figure 13. (a) Diffusive (blue) and turbulent (orange) fractions of the heat flux at the surface (\$y = 0\$), (b) Nusselt number for the cases of table 1.

3.3. Heat transfer and Reynolds analogy analysis

The contributions to the heat flux at the surface of the porous walls are now examined. To this end, the diffusive and turbulent components together with the dispersive component of the total heat flux q_{tot} , defined as

$$\frac{q_{tot}}{\rho c_p \Delta T} = \underbrace{\overline{\theta'v'}}_{q_{turb}/(\rho c_p \Delta T)} - \underbrace{\frac{q_{diff}/(\rho c_p \Delta T)}{\alpha_f}}_{\frac{d\Theta}{dy}}, \quad (3.1)$$

were calculated at the surface (\$y/\delta = 0\$) and are shown in figure 13(a). Since the dispersive heat flux is negligible, we denote $\overline{\theta'v'}$ as $q_{turb}/(\rho c_p \Delta T)$ (the turbulent heat flux). Figure 13(a) shows that, in the KP($\rangle$)b cases, the transport is mainly by turbulence since the diffusive heat flux accounts for less than 20% of the total heat flux at the surface. This causes $\overline{\theta'v'}$ to be considerably enhanced throughout the channel region (figure 12). The increased turbulent heat transfer in the channel region can be measured by the Nusselt number, defined as

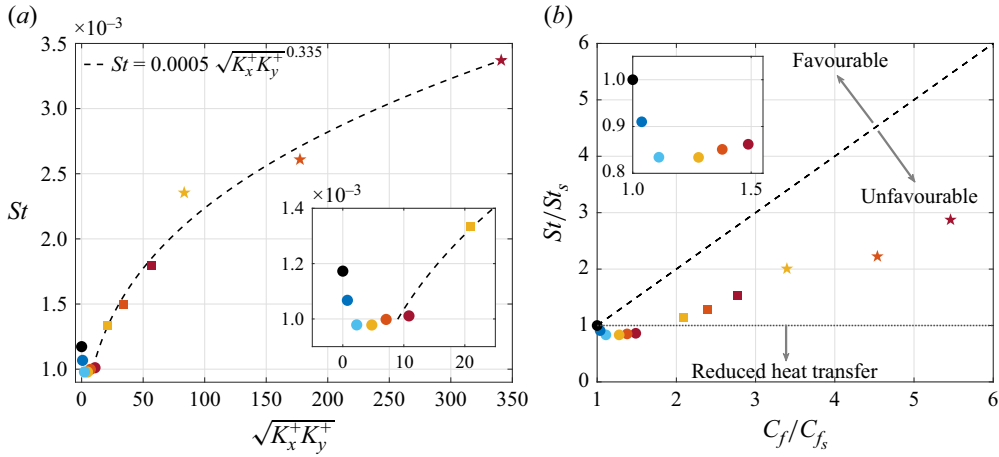


Figure 14. (a) Correlation of St against $\sqrt{K_x^+ K_y^+}$, (b) Reynolds analogy plot for the porous walls of table 1.

$$Nu = \frac{q_{tot} \delta}{\rho c_p \Delta T \alpha_f (\Theta_s - \Theta_b)}, \quad (3.2)$$

where $\Theta_b = \int_0^\delta U \Theta \, dy / \int_0^\delta U \, dy$ and $\Theta_s = \Theta(y=0)$ are the bulk mean temperature and the mean temperature at the surface, respectively. The Nusselt number values for all cases are gathered in figure 13(b), which serves as a companion plot to figure 13(a). It shows that a growing turbulent component in the total heat flux accompanies an increase in Nu and thus a more effective heat transport in the channel region.

As described in § 1, heat-transfer efficiency is technologically important. The porous walls act as passive structures, changing both the flow and heat-transfer characteristics of the system. Typically, heat transfer in systems involving convection is expressed in terms of the Stanton number, defined as

$$St = \frac{q_{tot}}{\rho c_p U_b \Delta T}. \quad (3.3)$$

Figure 14(a) shows the variation in heat transfer for the different porous walls by plotting St against $\sqrt{K_x^+ K_y^+}$. This structural parameter, introduced by Sharma *et al.* (2017) through linear stability analysis, was shown to predict the change in turbulence regimes from smooth-wall-like to K–H-like when applied to the DNS data of Habibi Khorasani *et al.* (2024). Hao & García-Mayoral (2025) also suggested this parameter as a key factor influencing the overlying turbulent flow, making it a suitable parameter for characterising porous walls. The heat transfer changes non-monotonically in figure 14(a). It initially decreases with $\sqrt{K_x^+ K_y^+}$, but increases as the flow transitions to the K–H-like regime.

Once in this regime, St continues to increase with $\sqrt{K_x^+ K_y^+}$, with St exceeding that of the baseline ZP from cases KP3a and beyond. A least-squares fit for the K–H-like cases gives the power-law relation $St = 0.0005 \sqrt{K_x^+ K_y^+}^{0.335}$, which is plotted in figure 14(a). This relation implies that St monotonically grows with $\sqrt{K_x^+ K_y^+}$ even at a very high porosity

of $\varphi = 0.95$, although less fast at large $\sqrt{K_x^+ K_y^+}$. However, we should expect that at higher porosity, St declines since the limit $\varphi = 1$ should resemble case *ZP* without a porous wall.

To further assess the heat-transfer efficiency, the approach of Rouhi *et al.* (2022) adopted from Bunker (2017) is followed and the ratio St/St_s versus C_f/C_{f_s} is examined. Here, St_s and C_{f_s} are the smooth-wall Stanton and skin-friction coefficient of case *ZP*. The skin-friction coefficient is defined as

$$C_f = \frac{2\tau_w}{\rho U_b^2}. \quad (3.4)$$

The Reynolds analogy with $2St_s/C_{f_s} \approx 1$ applies to the baseline smooth-wall case *ZP*. An unfavourable breakdown in the Reynolds analogy thus occurs if $St/St_s < C_f/C_{f_s}$, and means a lower heat-transfer efficiency since the amount of energy required to drive the flow is not matched by a proportional increase in heat transfer. In the opposite case of $St/St_s > C_f/C_{f_s}$, the breakdown is favourable and heat transfer becomes more efficient.

Plotting St/St_s against C_f/C_{f_s} in figure 14(b) shows that the breakdown in the Reynolds analogy is unfavourable for all present porous-wall cases. This finding differs from that of Motoki *et al.* (2022), who found the Reynolds analogy to hold in turbulent channel flow with permeable walls, even in the presence of K–H-type instabilities. However, they used the boundary condition of Jiménez *et al.* (2001) to produce the effect of a Darcy-type porous wall. This boundary condition assumes proportionality between pressure and wall-normal velocity fluctuations and was used along with no-slip and isothermal boundary conditions. As a result, conjugate heat-transfer effects and the flow within the porous wall were not accounted for, and the wall-normal velocity fluctuations at the porous surface were small, while the other velocity and temperature fluctuations were set to zero there. The flow over a modelled porous is likely different from that over a real porous wall (Camobreco *et al.* 2025). The initial decrease in heat transfer observed in figure 14(a) is also reflected in figure 14(b), which, coupled with the increase in C_f , translates into an increased flow resistance but reduced heat transfer for cases *SP*, *TP*, *KP3*, *KP2* and *KP1*. For the remaining cases St increases with C_f and does not attain a maximum limit.

Recall that, in § 1, it was mentioned that for rough-wall turbulent flows, MacDonald *et al.* (2019) observed that St for a constant roughness height of k/δ began to decrease with Re_b under fully rough-wall conditions, in line with the rough-tube experiments of Dipprey & Sabersky (1963). The latter concluded that ‘There is a limit for any combination of Reynolds number and Prandtl number beyond which increases in roughness, while increasing C_f , will no longer increase St ’. The data of Dipprey & Sabersky (1963) and MacDonald *et al.* (2019) were included by Rouhi *et al.* (2022) in their Reynolds analogy plot (figure 1 of their manuscript), showing this limit with a constant value of $St/St_s \approx 2$ beyond $C_f/C_{f_s} \approx 3$. However, this limit may depend on the density of the roughness (Abu Rowin *et al.* 2024) and can be higher for structured roughness. Turbulent heat transfer over ribs and dimpled surfaces can be approximately 3–4 times higher than over smooth surfaces (Webb *et al.* 1971; Han & Zhang 1992; García *et al.* 2012). A maximum limit is not reached in figure 14(b) beyond $C_f/C_{f_s} = 3$ for the porous walls. How far St/St_s can be increased before reaching the limit of a vanishing porous wall remains to be investigated.

3.4. Investigation of Reynolds analogy breakdown

In the preceding section, an unfavourable breakdown of the Reynolds analogy over the porous walls was established. The potential causes underlying this breakdown will now be investigated. For this purpose, we first examine the dissimilarity between momentum and heat transport in the channel. The turbulent Prandtl number, $Pr_T = \nu_T/\alpha_T$, is an

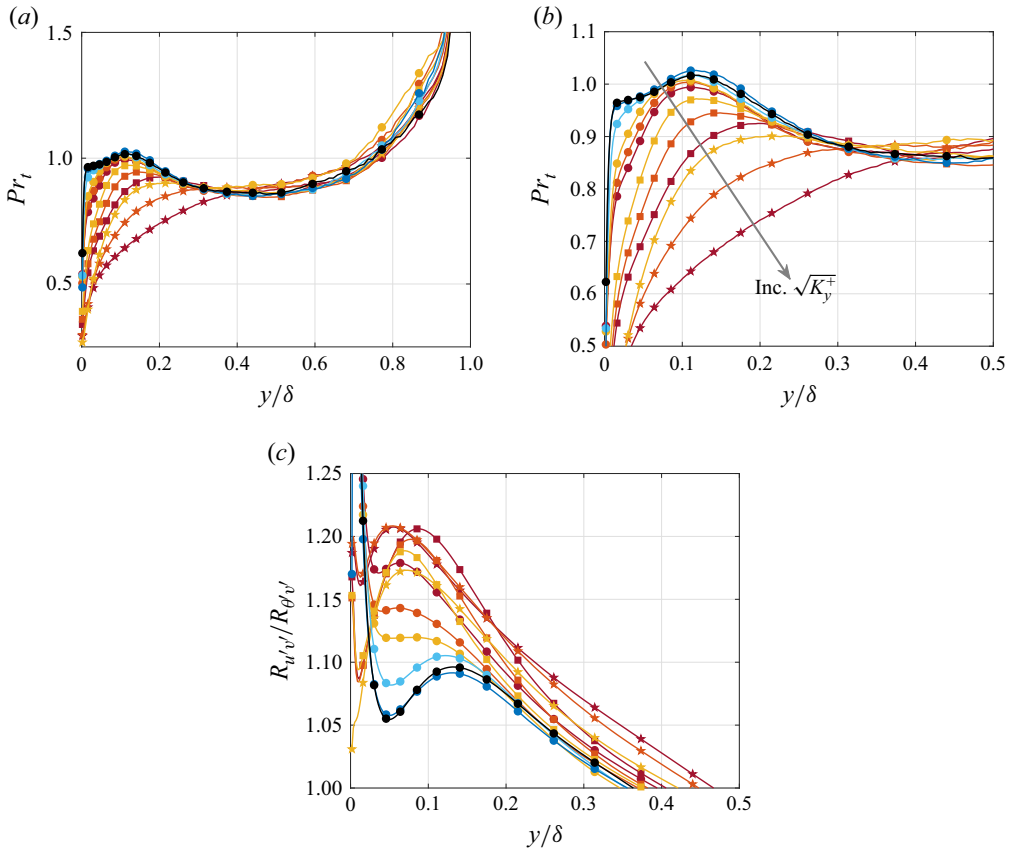


Figure 15. Turbulent Prandtl number (a) throughout the entire channel region and (b) the lower half of the channel region. (c) The ratio of the u' and v' correlation coefficient to the θ' and v' correlation coefficient in the lower half of the channel region.

indicator of this dissimilarity. Here, $\nu_T = \overline{u'v'}/(dU/dy)$ and $\alpha_T = \overline{\theta'v'}/(d\theta/dy)$ are the eddy viscosity and thermal eddy diffusivity, respectively.

Figure 15 shows Pr_T in the channel region. The profile in case ZP is similar to that reported for smooth-wall channel flows, with a peak value of $Pr_T \approx 1$ close to the surface and $Pr_T \approx 0.85$ farther away from it (see Pirozzoli, Bernardini & Orlandi 2016 and the references therein). The porous cases conform to the profile of ZP away from the surface, but closer to it differences emerge, with Pr_T deviating significantly when $0 < y/\delta < 0.2$. The deviations become greater and extend farther into the channel as the wall-normal permeability, $\sqrt{K_y^+}$, increases.

A decline in Pr_T indicates increasing dissimilarity between the turbulent transport of momentum and heat near the porous wall, with the thermal eddy diffusivity exceeding the eddy viscosity. Although, overall, the increase in heat transfer is smaller than the increase in skin friction for the present porous-wall cases (see § 3.3), the turbulent transport of heat is more effective than that of momentum close to the porous wall. By contrast, in turbulent heat transfer over the two-dimensional bar-type roughness of Leonardi *et al.* (2015), Pr_T was higher than the smooth-wall value at the roughness crests, going as high as 2.5, with one exception where Pr_T went as low as 0.5, presumably because that case did not belong to the fully rough regime. The surfaces of the present porous walls are like

perforated plates, with no elements protruding the surface level. The flow is therefore not subject to strong pressure drag and flow reversals at the surface as in flows over rough walls (Abu Rowin *et al.* 2024). This may partially explain why the momentum and heat-transfer dissimilarity over porous walls (figure 15*b*) differs from that over rough walls, and also why St/St_s does not level out like it does for rough-wall turbulence.

However, the present Pr_T results over porous walls align with those observed in canopy flows. Raupach, Finnigan & Brunet (1996) demonstrated that the turbulent flow and heat transport above a canopy layer conformed to that of a turbulent mixing layer with K–H-like rollers and similarly low values of $Pr_T \approx 0.5$. These K–H-like rollers also feature over the present porous walls and could explain the contradiction between the calculated Pr_T over the walls and the heat-transfer efficiency characterised through St/St_s vs. C_f/C_{fs} . Note also that low values of Pr_T are only observed above the surface where the K–H-like rollers have a large impact on turbulent transport. Further clarity is made by examining the correlation coefficient ratio

$$\frac{R_{u'v'}}{R_{\theta'v'}} = \frac{\overline{u'v'}/(u'_{rms}v'_{rms})}{\overline{\theta'v'}/(\theta'_{rms}v'_{rms})}, \quad (3.5)$$

in figure 15(*c*). The ratio grows larger near the surface as the wall becomes more permeable, with the position of the local maxima and minima moving closer to the surface. This reflects the growing dissimilarity between the turbulent heat and momentum flux and shows that the latter undergoes greater enhancement than the former, in agreement with the results of § 3.3.

In the present study, the turbulence dynamics thus affects the momentum and heat fields differently near the porous walls. To investigate this more deeply, the flow events contributing to $\overline{\theta'v'}$ and $-\overline{u'v'}$ at the surface are examined. This is done via quadrant analysis of $-\overline{u'v'}$ and $\overline{\theta'v'}$ using the joint probabilities of $(-u', v')$ and (θ', v') for select cases in figure 16. Since the temperature gradient has the opposite sign of the velocity gradient, events in the first (*Q1*) and third (*Q3*) quadrants of (θ', v') contribute positively to the wall-normal heat flux. The negative sign for u' in the joint probabilities of (u', v') is to therefore make the Reynolds shear stress generating events also correspond to *Q1* and *Q3*. It is evident when going from the smooth-wall-like case *SP* (figure 16*j*) to the K–H-like case *KP1* (figure 16*f*), that the overall distribution of events in the quadrant map changes, resulting in a greater number of high-intensity events in *Q1* and *Q3*. This trend continues in cases *KP1a* (figure 16*d*) and *KP1b* (figure 16*b*) with higher wall-normal permeability. A greater number of *Q3* events (large positive u' and large negative v') contribute to the generation of Reynolds shear stress, and the quadrant map has more asymmetry between *Q1* and *Q3* than between *Q2* and *Q4*. The quadrant maps for (θ', v') also undergo notable changes in shape when going from *SP* (figure 16*i*) to *KP1* (figure 16*e*), and become more diagonally symmetric. As a result, while contributions to $\overline{\theta'v'}$ increase in both *Q1* and *Q3*, there is no clear preponderance of either event type, with both quadrants largely mirroring each other. This is in contrast to the quadrant maps of $(-u', v')$, which have notable asymmetry between *Q1* and *Q3*. There are also many more events with extreme u' than θ' in the cases with high permeability. Events generating $-\overline{u'v'}$ are more numerous and intense than those generating $\overline{\theta'v'}$, and this dissimilarity in turbulent transport of heat and momentum may explain why the Reynolds analogy undergoes an unfavourable breakdown over porous walls.

This unfavourable breakdown contrasts with the local and global favourable breakdown of the Reynolds analogy seen in DNSs of turbulent flow and heat transfer over longitudinal slender triangular riblets and slender tall ribs, respectively (Kuwata 2022; Rouhi *et al.*

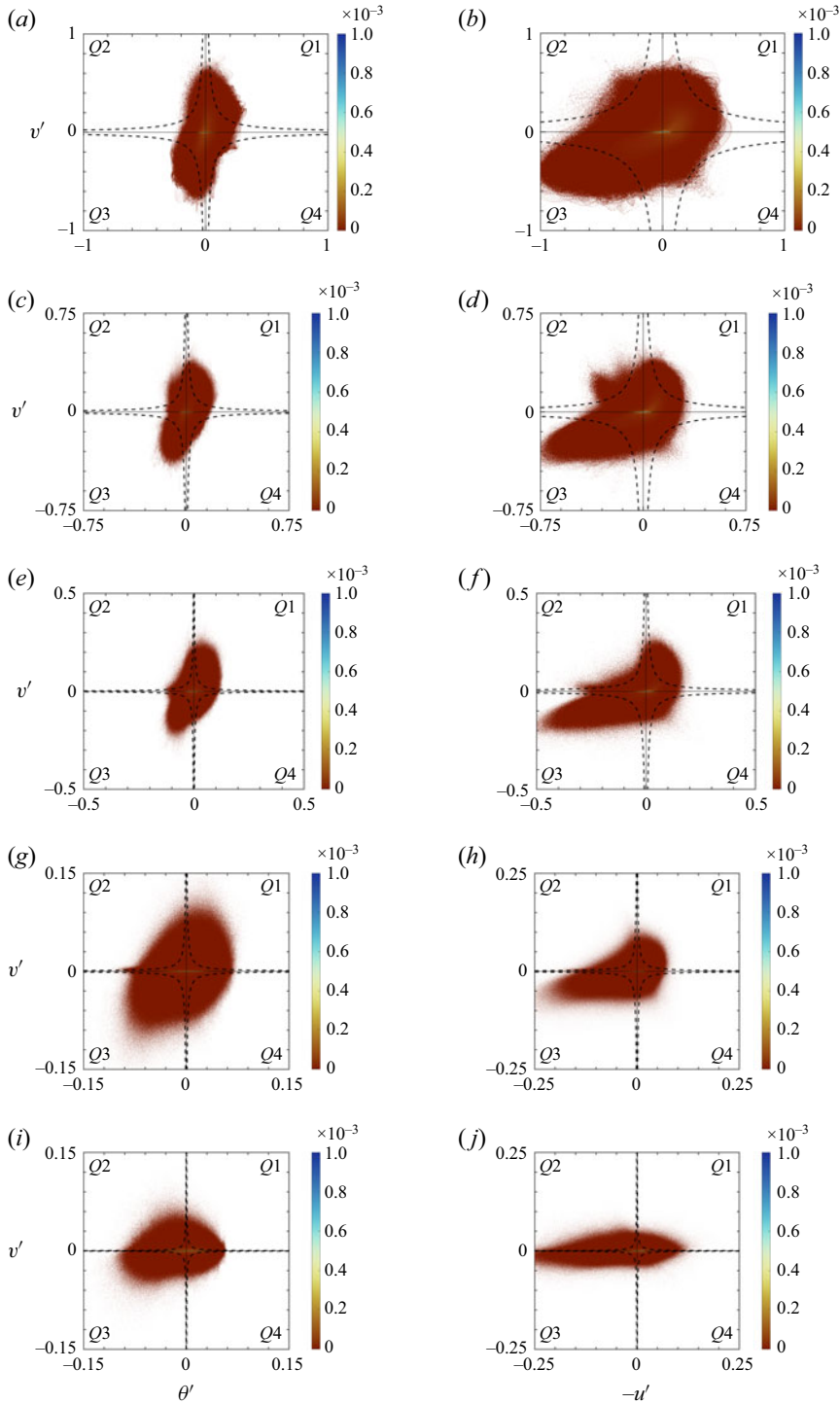


Figure 16. Quadrant analysis maps for (a, c, e, g, i) $\theta'v'$ and (b, d, f, h, j) $u'v'$ at $y=0$: (a,b) *KP1b*, (c,d) *KP1a*, (e,f) *KP1*, (g,h) *TP*, (i,j) *SP*. Dashed lines are hyperbolas corresponding to $|\theta'v'| = 8 \times \theta'v'$ and $|u'v'| = 8 \times -u'v'$. Empty bins are not shown and the velocity fluctuations are normalised by U_b . Note that the range of the axes is not the same in all the plots.

2022). In flows over triangular riblets, the local favourable breakdown was driven by strong K–H-like rollers that produced patches of negative wall shear stress within the grooves. Since these reversals were not induced by pressure drag (as over roughness), the heat transfer (being strictly positive) locally exceeded the momentum transfer (Rouhi *et al.* 2022). In the case of longitudinal ribs, the favourable breakdown and strongly enhanced heat transfer observed for certain rib spacings arose from pressure variations generated by intense K–H-like rollers, which reduced turbulent momentum more than heat transfer (Kuwata 2022).

In the present study, K–H-like rollers are likewise associated with increased heat transfer, yet the Reynolds analogy breaks down unfavourably. A likely contributing factor is conjugate heat transfer. The porous substrate in our DNSs has a temperature close to that of the surrounding fluid, so the substrate surface is much cooler than the bottom wall and the heat transfer from the fluid to the substrate is limited. By contrast, in Rouhi *et al.* (2022) and Kuwata (2022), much of the fluid–solid heat exchange occurs at the tops of the riblets/ribs, which are held at a uniform temperature. Such exchange would likely be constrained if conjugate effects were included, but these were not accounted for in those studies.

Additional factors may contribute to the unfavourable (porous walls) versus favourable (riblets/ribs) Reynolds analogy outcomes and deserve further investigation. We note, however, that for riblets/ribs, the core of the K–H-like rollers, with strong pressure variations, is in the shear layer near the textured interface, enabling the local/global favourable breakdown (Kuwata 2022; Rouhi *et al.* 2022). In the present DNSs, much of the fluid–solid heat exchange occurs at the bottom wall, whereas the shear layer, and thus the roller cores, lies above the porous interface, farther from the bottom wall. Consequently, the pressure variations near the bottom wall are likely too weak to induce a favourable Reynolds analogy breakdown.

4. Summary and discussion

Direct numerical simulations of porous-wall turbulence with conjugate heat transfer have been conducted in this study. One focus of the study has been to assess the heat-transfer performance over porous walls compared with other passive structures such as rough walls. Unlike rough walls, which experience a limit once in the fully rough regime, the heat transfer over porous walls continuously grows with permeability even up to a very high porosity of $\varphi = 0.95$. The equivalent of a form-drag-dominating regime does not develop over a porous wall, which is the main mechanism behind the limitation of heat transfer over rough walls. Under fully rough conditions, the diffusive mechanism becomes so diminished across the elements of a rough surface that the heat flux becomes limited, whereas the flow resistance becomes dominated by form drag. This situation does not occur at the surface of a porous wall. Unlike a rough wall, a porous structure permits wall-normal flow penetration, which prevents flow separation. This is of course highly dependent on the interfacial topography of a porous wall. The walls examined in this work have no protruding elements at the surface, minimising their interfacial roughness. For other surface topographies, form drag may play a role. Nevertheless, for porous walls an upper limit for heat transfer must also exist, since ultimately the heat-transfer rate must approach that over a smooth wall when the permeability is further increased and approaches unity. This limit is not yet reached here.

To further study the heat transfer, the ratio St/St_s versus C_f/C_{f_s} was examined. This quantifies the fractional increases in heat transfer against the fractional increases in skin friction relative to a smooth-wall turbulent flow. In common with almost all rough surfaces,

porous walls cause an unfavourable breakdown of the Reynolds analogy. Turbulent heat transfer over random roughness saturates at $St/St_s \approx 2$ (Dipprey & Sabersky 1963; Rouhi *et al.* 2022), although this maximum can still depend on the roughness density (Abu Rowin *et al.* 2024), while over manufactured roughness such as ribs and dimples it can reach higher maxima of $St/St_s \approx 3 - 4$ (Webb *et al.* 1971; Han & Zhang 1992; García *et al.* 2012). Over the present porous walls it becomes as high as $St/St_s \approx 3$ with no limit yet reached. In the low-permeability porous-wall cases with smooth-wall-like turbulent flow, an increasing permeability increases flow resistance but reduces heat transfer since a higher permeability means less solid material with a higher thermal conductivity. At higher permeability the flow enters the K–H-like regime and the heat transfer grows with permeability. The permeability at which the heat-transfer trend changes from decreasing to increasing likely depends on the thermal properties of both the solid and fluid.

Although the breakdown of the Reynolds analogy is unfavourable, the turbulent Prandtl number, Pr_T , decreases with permeability and drops below unity in the region above the porous surface, showing that the thermal eddy diffusivity exceeds the eddy viscosity there. Canopy flows, which share features with the present flows such as the spanwise K–H-like rollers, show similarly low values of Pr_T . Thus, immediately above the porous wall, turbulent heat transport is more efficient than turbulent momentum transport, however, overall the skin friction increases faster than heat transport with permeability due to the influence of the porous wall. Quadrant analysis of turbulent shear stress and heat flux reveals that high intensity events feature much more prominently in the generation of Reynolds shear stress than in turbulent heat flux. This disparity increases as the wall permeability increases.

Although the present investigation covered a wide range of permeabilities, it was limited to a single type of porous structure composed of square bars. The depth of the porous substrate and the Reynolds number of the flow were also kept constant. Further studies are needed to determine whether the observed enhancement of heat transfer and the breakdown of the Reynolds analogy can be generalised to other porous structures, fluids and Reynolds numbers.

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