

Local slip length and surfactant effects on liquid-infused surfaces

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Robust surfaces capable of reducing flow drag, controlling heat and mass transfer, and resisting fouling in fluid flows are important for various applications. In this context, textured surfaces impregnated with a liquid lubricant show promise due to their ability to sustain a liquid–liquid interface that induces slippage. However, theoretical and numerical studies suggest that the slippage can be compromised by surfactants in the overlying fluid, which contaminate the liquid–liquid interface and generate Marangoni stresses. In this study, we use Doppler-optical coherence tomography, an interferometric imaging technique, combined with numerical simulations to investigate how surfactants influence the slip length of lubricant-infused surfaces with longitudinal grooves in a laminar flow. Surfactants are endogenously present in the contrast agent (milk) which is added to the working fluid (water). Local measurements of slip length at the liquid–liquid interface are significantly smaller than theoretical predictions for clean interfaces (Schönecker & Hardt 2013). In contrast, measurements are in good agreement with numerical simulations of fully immobilized interfaces, indicating that milk surfactants adsorbed at the interface are responsible for the reduction in slippage. This work provides the first experimental evidence that liquid–liquid interfaces within textured surfaces can become immobilised in the presence of surfactants and flow.

Key words: wetting and wicking, drops, capillary flows

1. Introduction

Lubricant-infused surfaces (LISs) submerged in liquid flows hold significant potential for drag reduction (Solomon, Khalil & Varanasi 2014; Rosenberg *et al.* 2016), biofouling

resistance (Epstein *et al.* 2012) and enhanced heat transfer (Sundin *et al.* 2022). These surfaces are characterized by a structured solid texture infused with a lubricant that is immiscible with the surrounding fluid. Lubricant-infused surfaces are capable of self-healing and are not susceptible to failure induced by hydrostatic pressure. Therefore, LISs offer greater robustness than superhydrophobic surfaces (SHSs) in submerged environments when designed appropriately (Wong *et al.* 2011; Wexler, Jacobi & Stone 2015; Preston *et al.* 2017; Sundin, Zaleski & Bagheri 2021).

The beneficial properties of LISs rest upon the existence of a stable and mobile liquid–liquid interface, which contributes to a slip effect on the external flow. The main factors that can impede the efficacy of LISs are lubricant drainage and the immobilization of the interface. The former is a recognized issue – initially identified by Wexler *et al.* (2015) – caused by prolonged exposure of a LIS to shear flow. Viscous stresses can deform the fluid interface, displacing the lubricant. For a LIS with a pattern of grooves parallel to the flow direction, a potential solution to prevent lubricant drainage is to confine the lubricant in cavities or segments (Fu *et al.* 2019) which are shorter than the maximum retention length prescribed by Wexler *et al.* (2015). An additional retention mechanism has been demonstrated by Saoncella *et al.* (2024), which leverages the high contact angle hysteresis between the solid and the lubricant to stabilize lubricant droplets within the grooves when subjected to shear flow.

The second limiting factor (interface immobilization) can be induced by surface-active agents (surfactants) adsorbed at the liquid–liquid interface. While this phenomenon has been studied for SHSs, there is still a lack of experimental investigations to characterize this immobilization on LISs. The investigations conducted for SHSs have demonstrated that the presence of surfactants at the gas–liquid interface can significantly reduce the slippage. The reduced slip on SHSs is caused by small gradients in concentration of surfactants adsorbed at the interface, which generate a Marangoni stress opposing the flow (Bolognesi, Cottin-Bizonne & Pirat 2014; Peaudecerf *et al.* 2017; Song *et al.* 2018). Theoretical and numerical models support these observations and provide additional insights into the underlying mechanisms (Landel *et al.* 2019; Temprano-Coleto *et al.* 2023; Baier 2023).

A recent numerical work by Sundin & Bagheri (2022) used an adsorption model for alkane–water interfaces (Fainerman *et al.* 2019) to investigate effects of surfactants on LISs consisting of transverse grooves and subjected to a shear flow. Based in the surfactant theory for SHSs (see e.g. Landel *et al.* (2019)), the authors developed an analytical model that predicts the effective slip length of the LIS in relation to the concentration of surfactants in the working fluid. Their findings demonstrate that the Marangoni stress significantly reduces the slip length, and highlight the sensitivity of the liquid–liquid interface to contaminants. This reinforces the necessity for experimental investigations, which are currently lacking.

It is recognized that surface-active contaminants represent a significant challenge in experimental investigations involving interfacial flows and surfaces. For example, PDMS (polydimethylsiloxane) – a common material used to fabricate microfluidic channels – releases un-cross-linked oligomer chains (Wong *et al.* 2020), which are surface-active (Hourlier-Fargette *et al.* 2017) and diffuse into the solution (Regehr *et al.* 2009; Carter *et al.* 2020). Peaudecerf *et al.* (2017) and Sundin & Bagheri (2022) report that trace amounts of surfactants in the bulk is sufficient to reduce the slip length, e.g. bulk concentration of 10^{-4} mol m⁻³ for SHS and 10^{-5} mol m⁻³ for LIS results in a 50 % slip length reduction.

This study aims to quantify the slip length of LISs with longitudinal grooves to better understand the effects of surfactants. Doppler-optical coherence tomography (D-OCT), an interferometric imaging technique, is employed to measure the local velocity profile in a

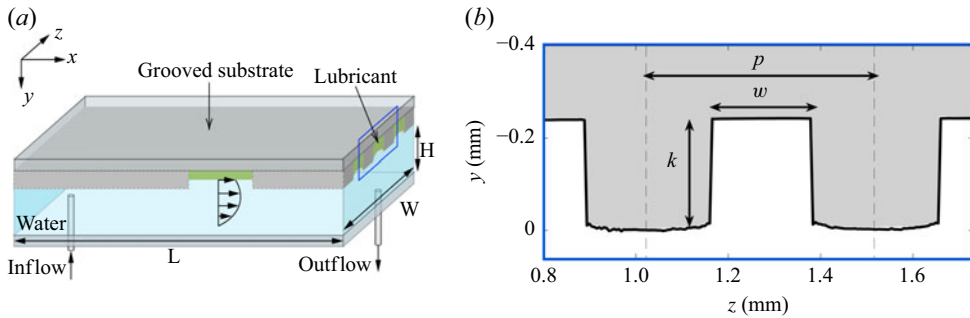


Figure 1. (a) Schematic representation of the experimental set-up. The proportions of the sketch are not on a real scale. (b) Profile (measured using optical coherence tomography (OCT)) of the cross-section of a solid substrate used for the LIS. The dimensions represented are the groove width, w , the groove depth, k , and the pitch p . This portion of grooved profile is located on the top wall of the duct, in the area highlighted by a blue rectangle in (a).

laminar duct with a LIS mounted on one wall. The OCT device is also used to extract the interface shape. By complementing the experiments with targeted numerical simulations, we show that surfactants can nearly fully rigidify the liquid–liquid interface.

The structure of this paper is as follows. Section 2 describes the flow configuration and the experimental methodology. Section 3 presents the experimental results, comparing the local slip length with analytical models of LISs with and without surfactants. Section 4 discusses the numerical results and their comparison with experiments, providing indirect evidence that surfactant contamination is responsible for the reduced slippage observed experimentally. Finally, § 5 summarizes the paper and discusses its main outcomes.

2. Configuration and methods

This section presents the flow configuration and the design of LISs used in the experiments. The methodology and the scheme used to measure the shape of the liquid–liquid interface and the flow velocity are also described.

2.1. Flow cell and LIS design

Figure 1(a) shows the flow channel used in this study. The height of the main chamber of the duct is delimited by the coordinates $y = 0$ and $y = H$, with $H = 770 \mu\text{m}$, the width is $W = 3.25 \text{ mm}$ and the length is $L = 80 \text{ mm}$. The top surface is patterned with rectangular grooves parallel to the flow direction, which constitute the solid substrate of the LIS. The inlet and outlet of the flow are located at the extremities of the bottom wall through holes of 1 mm in diameter. The detailed description of the channel assembly is reported in Appendix A.1. Hexadecane (Sigma–Aldrich), with dynamic viscosity $\mu_l = 3.1 \text{ mPa s}^{-1}$, is employed as a lubricant to impregnate the grooves, while the working fluid consists of an aqueous solution of glycerol–water mixed with milk at 20 % volume ratio (later referred to as simply water) with dynamic viscosities ranging from $\mu_w = 0.9 \text{ mPa s}^{-1}$ to 40.7 mPa s^{-1} depending on the glycerol ratio. The viscosity ratio between water and lubricant is defined as $N = \mu_w / \mu_l$. Note that the lubricant infused in the grooves is located at coordinates $y < 0$.

A total of five experimental cases with different geometrical features and viscosity ratio were designed (see table 1). Three grooved surfaces, differing by the groove aspect ratio, are fabricated in transparent resin using a casting technique (see Appendix A.2).

Case	$w[\mu\text{m}]$	$k[\mu\text{m}]$	$p[\mu\text{m}]$	A	a	N	ϕ
A059N03	376 ± 5	223 ± 3	621 ± 2	0.59	0.61	0.3	$32.1^\circ \pm 1.2^\circ$
A089N03	271 ± 5	242 ± 3	608 ± 2	0.89	0.45	0.3	$20.6^\circ \pm 2.9^\circ$
A106N03	223 ± 5	237 ± 3	495 ± 2	1.06	0.45	0.3	$24.6^\circ \pm 2.3^\circ$
A106N1	223 ± 5	237 ± 3	495 ± 2	1.06	0.45	1.2	$21.2^\circ \pm 1.7^\circ$
A106N13	223 ± 5	237 ± 3	495 ± 2	1.06	0.45	13.2	$29.8^\circ \pm 2.9^\circ$

Table 1. The parameters defining the LISs for the five configurations (see also Figure 1): groove width w , groove depth k , groove pitch p , aspect ratio $A = k/w$, slipping surface fraction $a = w/p$, viscosity ratio $N = \mu_w/\mu_l$ and protrusion angle of the meniscus ϕ .

Case	$\beta^{Sch}(0) [\mu\text{m}]$	$\langle \beta^{int} \rangle [\mu\text{m}]$	R	$\tau_l \cdot 10^{-3} [\text{Pa}]$	$\tau_w [\text{Pa}]$	τ_l/τ_w
A059N03	34.0	5.1 ± 1.5	6.8 ± 2.0	1.5 ± 0.2	0.0057 ± 0.0001	0.26 ± 0.04
A089N03	28.1	4.7 ± 0.5	5.9 ± 0.6	1.4 ± 0.2	0.0055 ± 0.0002	0.26 ± 0.04
A106N03	23.2	4.6 ± 0.5	5.0 ± 0.5	1.6 ± 0.2	0.0052 ± 0.0002	0.31 ± 0.05
A106N1	77.3	6.8 ± 0.7	11.3 ± 1.2	2.2 ± 0.2	0.0186 ± 0.0003	0.12 ± 0.01
A106N13	1004.9	12.7 ± 1.0	78.9 ± 6.2	3.0 ± 0.3	0.1917 ± 0.0036	0.015 ± 0.002

Table 2. Central slip length value predicted by Schönecker & Hardt (2013), $\beta^{Sch}(0)$, measured average slip length over the lubricated groove, $\langle \beta^{int} \rangle$, ratio $R = \beta^{Sch}(0)/\langle \beta^{int} \rangle$, estimated shear stress from conserved (i.e. closed cavity) lubricant τ_l , measured shear stress at the water–lubricant interface τ_w and their ratio.

An example of a groove profile is shown in figure 1(b). The grooves have depth k of approximately $230 \mu\text{m}$, while their widths w vary from 223 to $376 \mu\text{m}$, resulting in aspect ratios $A = k/w$ ranging from 1.06 to 0.59 . The slipping area fraction, defined as $a = w/p$, ranges between 0.45 and 0.61 . The influence of the viscosity ratio on the slip length was tested using the LIS with aspect ratio $A = 1.06$. According to theoretical models, the effective slip length can vary significantly with both groove aspect ratio A and the groove slipping fraction a (Schönecker *et al.* 2014). However, for the LIS geometries considered here, the effective slip depends weakly on A and a , i.e. we do not investigate shallow grooves ($A < 0.5$) or very thin ($a < 0.2$) or wide slits ($a > 0.8$). We merely investigate different geometries to ensure that our observations are not specific to one particular LIS geometry. More specifically, as shown in Schönecker *et al.* (2014), the effective slip length increases with a since the interface relative to solid increases, allowing for larger slip. The effective slip length also increases with A as the viscous dissipation in the lubricant is reduced with deeper grooves, until around $A = 1$, where the velocity gradients of the lubricant near the interface become independent of the flow deep in the groove. For our configurations: $A = 0.59$, $A = 0.89$ and $A = 1.06$, with $a = 0.61$, $a = 0.45$ and $a = 0.45$, respectively, the theoretically predicted effective slip lengths (reported in second column of table 2) show a modest variation.

The water solution was mixed with glycerol in mass ratios of $1:0.7$ and $1:3.5$, increasing the dynamic viscosity of the water–glycerol solutions to 3.6 mPa s^{-1} and 40.7 mPa s^{-1} , respectively (computed using the empirical formulae of Cheng (2008) and Volk & Kähler (2018)). Accordingly, the viscosity ratio N is increased from 0.3 to 1.2 and 13.2 . As detailed in Appendix A.2, the LIS is prepared by first filling the duct with lubricant and then with the water solution at creeping flow. The lubricant remains trapped inside the grooves by capillary forces. After complete filling of the duct, the flow rate was gradually increased to 0.4 mL min^{-1} , which provides a bulk velocity $U_b = 2 \text{ mm s}^{-1}$ and

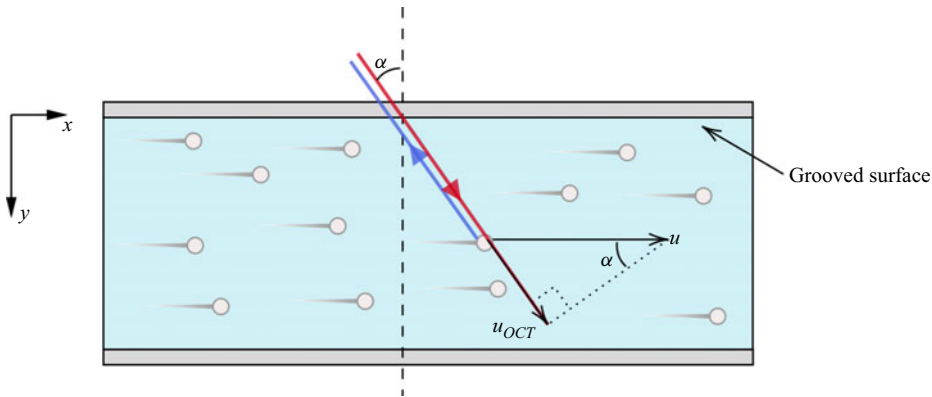


Figure 2. Schematic of velocity components obtained through D-OCT. The phase of the incoming reference beam (red) is compared with that of the backscattered beam (blue) from the moving particle to obtain the parallel component of the particle velocity.

a Reynolds number $Re \approx 1$. The imposed velocity is a compromise between having a high D-OCT resolution (higher resolution at higher velocities) while maintaining the stability of the lubricant-filled groove.

2.2. Optical coherence tomography

Optical coherence tomography is a low-coherence interferometry-based technique used to generate images that capture optical scattering within opaque samples on a millimetre scale, with a resolution of just a few micrometres. While OCT has primarily been used in the medical field (Yonetsu *et al.* 2013), recently it has been employed also in microfluidics and soft-matter research (Haavisto *et al.* 2015; Gowda *et al.* 2019; Lauri *et al.* 2021; Huisman *et al.* 2024; Gupta *et al.* 2024; Lauga *et al.* 2005; Amini *et al.* 2025).

The functioning principle of OCT is based on a Michelson interferometer. A broadband light source is split into two components using a beam splitter. One component is directed into a reference arm, while the other is transmitted through the sample. The light reflected from the sample is recombined at the beam splitter with the reference beam, producing an interference signal. When the optical-path-length difference is within the coherence length of the light source, interference occurs (Tomlins & Wang 2005). The output spectrum is analysed by a spectrometer, generating a complete interference pattern that represents variations in intensity as a function of depth. Within this pattern, refractive index variations between layers in the sample manifest as intensity magnitude variations, providing detailed information about the sample's depth.

When there is a flow, the OCT imaging technique can be combined with the acquisition of a Doppler frequency (D-OCT) to obtain velocimetry information (Chen *et al.* 1997). The interference of light backscattered from a moving particle with the reference beam produces a beating at the Doppler frequency, which generates a corresponding Doppler phase shift. This phase shift is proportional to the projected component of the velocity measured along the OCT beam, u_{OCT} , as illustrated in figure 2. To reconstruct the velocity component in the x (streamwise) direction, an angle α between the beam and the vertical direction is required, yielding $u = u_{OCT} / \sin \alpha$. The velocity resolution depends on the acquisition time and the scan angle. We used a Telesto II Spectral Domain OCT apparatus (Thorlabs Inc., NJ, USA), which has a central wavelength of 1310 nm and a nominal bandwidth of 270 nm, leading to a depth resolution of 2.58 μm in water.

The measurement of u_{OCT} is related to the doppler phase shift

$$u_{OCT} = \frac{\lambda f \Delta\varphi}{4\pi} \quad (2.1)$$

where $\lambda = 1310 \text{ nm}$ is the central wavelength of the infrared source, $f = 5.5 \text{ kHz}$ is the scanning frequency and $\Delta\varphi \in [-\pi, \pi]$ (rad) is the phase difference between two consecutive acquisitions, corresponding to the Doppler shift. This leads to the maximal measurable velocity $u_{OCT,max} = \lambda f / 4 = 7.2 \text{ mm s}^{-1}$. For more details, see Amini *et al.* (2025).

In these experiments, commercial milk (lactose-free pasteurized semiskimmed cow milk: Arla Ekologisk Lactofri Mellanmjölkdryck 1.5 % fett) is added to water at a volume ratio of 1 : 5 as an infrared diffusing contrast agent for the OCT/D-OCT measurement. No significant change in the viscosity, interfacial tension or refractive index of water is measured at this concentration of milk. As milk mixes with water rather than with the lubricant, the contrast in the intensity signal allows the two fluid components to be clearly distinguished. Optical coherence tomography is used here to image the LIS inside the duct during flow and in Doppler mode for the acquisition of velocity profiles. The flow cell is positioned below the OCT objective at an angle $\alpha \approx 6^\circ$. The OCT beam enters the duct through the clear top wall.

2.2.1. Intensity measurements

The intensity signal acquired with OCT across the flow cell allowed us to monitor the distribution of lubricant within the grooves during the initial infusion and the subsequent measurements. As an example, [figure 3](#) shows a cross-section of the flow cell during flow with the LIS mounted at the top wall (highlighted with a dashed line) and lubricant within the grooves. From this tomogram, it can be seen that the water–lubricant interface is curved with a meniscus bending towards the top. This is due to a pressure difference across the interface and to the hydrophobicity of the grooved wall. The shadows in the working fluid below the vertical walls of the grooves are due to the extreme grazing angle between the OCT beam and these vertical structures. These regions are excluded from the measurements in the following. The deflection ϕ of the liquid interfaces from the level $y = 0$ was measured from the tomograms in streamwise positions corresponding to 4.5, 5.0, 5.5, 6.0 and 6.5 cm from the duct inlet. For each case, the streamwise average of the deflection and the associated standard deviation were calculated and reported in [table 1](#). In the following, it is assumed that the interface is of constant curvature $\kappa = 2 \tan \phi / w$ in the spanwise direction, and zero curvature in the streamwise (longitudinal) direction.

2.2.2. Velocity measurements

The measurements of velocity profiles were conducted through the full depth of the duct at the groove closest to the axial centre of the duct, where the influence of sidewalls on the flow is minimal. The acquisition frequency was 5.5 kHz. The locations of the local measurements are shown in the insert of [figure 3](#) where they are marked by blue-filled circles at the water–solid boundary and by green circles at the water–lubricant interface. The spanwise spacing between them is $30 \mu\text{m}$. For each configuration, five spanwise series of velocity profiles straddling a pitch were recorded in the streamwise locations corresponding to 4.5, 5.0, 5.5, 6.0 and 6.5 cm from the duct inlet, where the flow was fully developed. The results presented are obtained from a single experiment performed for each configuration.

The velocities measured in the vicinity of the groove, normalized with the bulk velocity U_b , are shown as a contour plot in [figure 4\(a\)](#) for case A106N13. Note that the y -axis is

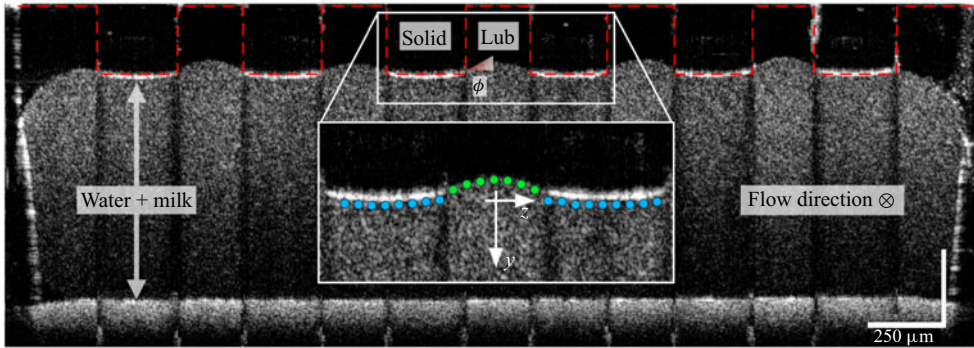


Figure 3. Cross-sectional scan of the duct acquired with OCT at streamwise position corresponding to 4.5 cm from the inlet. The grooves, marked as ‘solid’ are mounted on the top wall and their profile is highlighted with a red dashed line. The space between the grooves is filled with lubricant, marked with ‘lub’, whose profile bows at an angle ϕ to the top of the grooves. The volume of the duct is filled with a water–milk mixture which appears opaque. The insert shows an enlargement of the measurement positions marked with blue dots at the water–solid interface and with green dots at the water–lubricant interface.

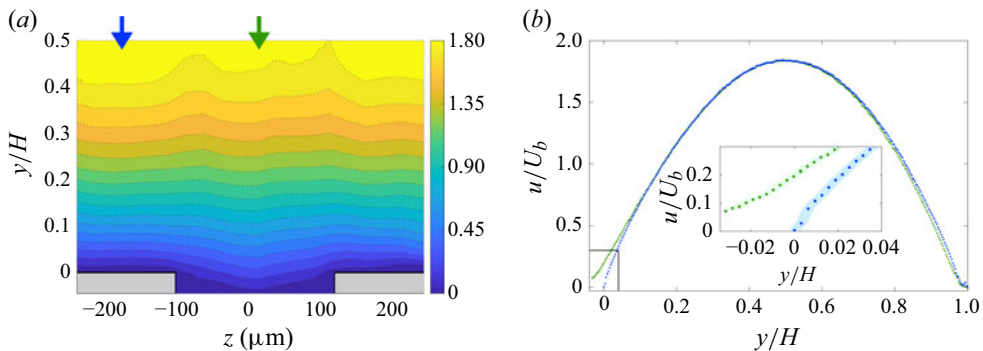


Figure 4. (a) Contour plot of the velocity measurements in the central region of the duct for case A106N13. The arrows indicate the location of the measurements shown in (b). (b) Velocity profiles were acquired at the water–lubricant interface (green markers) and the water–solid interface (blue markers), for the same case. In the insert, the coloured area surrounding the markers represents the standard deviation of the velocity.

now directed upwards and the grooved surface is at the bottom. The black solid lines represent the solid ridges, while the blue and green arrows show the locations of the velocity profiles displayed in figure 4(b). The two velocity profiles in figure 4(b) are shown using the same colour code as the arrows in figure 4(a). The top and bottom walls of the flow cell have coordinates $y = 0$ and $y = H$, respectively. They both peak at a normalized velocity $u/U_b = 1.83$ which is compatible with the viscous flow in a rectangular channel of section $770 \mu\text{m} \times 3.25 \text{ mm}$ where $u_{th}/U_b = 1.84$ is predicted Panton (1984) and Amini *et al.* (2025). The velocity profile measured on the solid ridge is analogous to a Poiseuille flow, exhibiting zero velocity at the walls. In contrast, the profile measured on the liquid–liquid interface deviates from the Poiseuille profile due to the presence of slip at the interface. The increased distance from the top wall of the green profile is due to the interface protruding towards the groove bottom. In the insert of figure 4(b), an enlargement of the velocity profiles shows the uncertainty intervals of the measurements calculated as the standard deviation which, near the boundary, equals to $\pm 5\%$ of the measured value. In the following, the key parameter extracted from the

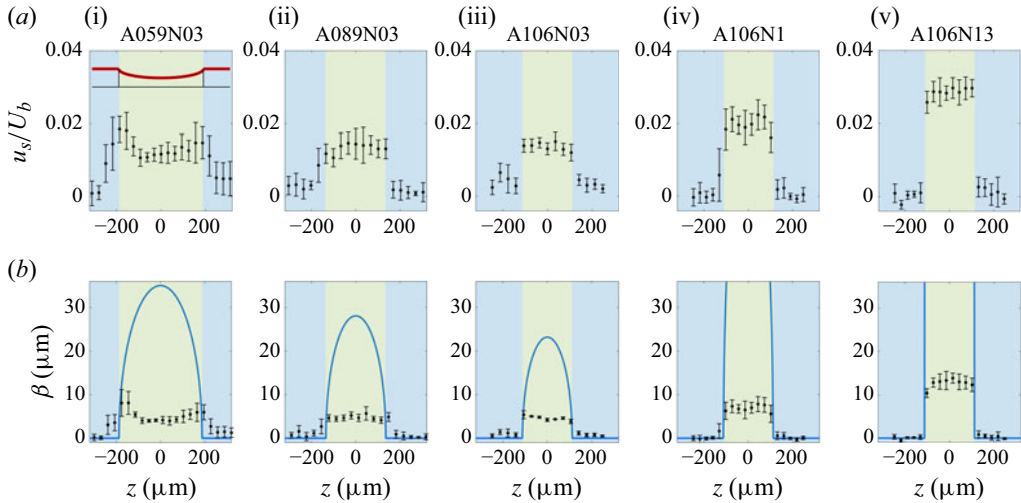


Figure 5. See table 1 for geometrical parameters: (a) streamwise-averaged slip velocities as a function of the spanwise position z . The error bars correspond to the standard deviations. The sketch in (i) indicates that the values are calculated at the interface. (b) Local slip lengths obtained from measurements, $\beta^{int}(z)$ (markers) and from the analytical model of Schönecker & Hardt (2013), $\beta^{Sch}(z)$ (solid line). In (iv) and (v), the curve corresponding to the model exceeds the boundary of the plot. The background colour of the tiles identifies the region of the liquid–liquid interface (green) and the region of the liquid–solid interface (blue).

velocity measurements is the streamwise-averaged interfacial slip velocity $u_s(z)$, which depends on the z position due to the interface bending.

3. Slip of LIS

This section presents the experimental results and provides a comparative analysis with analytical predictions. The difference between the experiments and theory is discussed taking into account the influence of contaminants on the mobility of the liquid–liquid interface.

3.1. Local slip length

Figure 5(a) shows the streamwise-averaged slip velocities $u_s(z)$ normalized by the bulk velocity U_b , measured along both the liquid–solid and liquid–liquid interfaces within a single pitch. Figure 5(a i–iii) correspond to cases with increasing aspect ratio and constant viscosity ratio (A059N03, A089N03 and A106N03). Figure 5(a iv,v) represent configurations with the same geometry as figure 5(a iii) but with higher viscosity ratios (A106N1 and A106N13). The green background represents the physical width of the groove – containing the liquid–liquid interface – while the blue colour represents the area occupied by the liquid–solid interface. In all configurations, the slip velocity at the solid boundary is approximate 0.2 % of U_b . Given the accuracy of our measurements, this is compatible with the expected no-slip condition there. At the liquid interface, the slip velocity averages 1.3 % of U_b for the cases with low viscosity ratio $N = 0.3$, increasing to 2 % and 3 % with increasing viscosity of the water phase, with $N = 1$ and $N = 13$, respectively. The variance associated with the slip velocities are calculated as the standard deviation of the repeated scans, one spanwise set per each of the five streamwise positions, whose average gives a single velocity measurement.

For each velocity profile, the ratio between the slip velocity and the velocity gradient $\partial u / \partial y$ is calculated and the local slip length at the interface, $\beta^{int}(z)$, is

evaluated as the average of the repeated measurements at n streamwise positions, ($x_j = 4.5, 5.0, 5.5, 6.0, 6.5$ cm),

$$\beta^{int}(z) = \frac{1}{n} \sum_{j=1}^n \frac{u_s(z, x_j)}{\frac{\partial u(z, x_j)}{\partial y}|_{interface}}. \quad (3.1)$$

The superscript ‘*int*’ denotes variables evaluated at the curved liquid–liquid interface, corresponding to the red curve in the sketch in [figure 5\(a i\)](#). The variation in slip velocity between the solid and liquid boundaries translates directly to the corresponding slip length values shown in [figure 5\(b\)](#). At the liquid–solid interface, the measured slip length remains consistently around $1 \pm 1 \mu\text{m}$ (no-slip condition) across all cases. In contrast, the slip length at the liquid–liquid interface is $5 \pm 1 \mu\text{m}$ for the first three cases, increasing to $7 \pm 2 \mu\text{m}$ and $13 \pm 2 \mu\text{m}$ for the fourth and fifth cases, respectively. The uncertainty associated with the local slip length is the standard deviation of the repeated measurements in the different streamwise positions (see [Appendix B](#)).

To understand how the slip length of a realistic LIS set-up differs from an ideal theoretical configuration, we compare our results to the local slip length predicted by the model of Schönecker & Hardt (2013), here named $\beta^{Sch}(z)$. Their analytical equation characterizes the variation along the pitch of the local slip length of a shear flow over an infinitely long rectangular groove aligned with the flow direction. The cavity contains a second fluid with different viscosity, and the interface between the two fluids is flat. The local slip length is modelled as an elliptic function with a maximum value at the centre of the groove (where the flow is least affected by the sidewalls of the cavity itself), expressed by the dimensionless variable D . These expression reads

$$\beta^{Sch}(z) = \text{Re} \left\{ -i2ND \sqrt{z^2 - \frac{w^2}{4}} \right\} \quad (3.2)$$

$$D = d_0 \times \text{erf} \left(\frac{\sqrt{\pi}}{2d_0} A \right) \quad (3.3)$$

where Re stands for the real part, $d_0 = 0.347$ and erf is the error function. The modelling of D is discussed in depth by Schönecker & Hardt (2013) and is based on previous investigations of lid-driven cavity flows. Note that, in their consecutive work (Schönecker *et al.* 2014), the analytical expression of the local slip length distribution was derived for an array of parallel lubricant-filled grooves, which accounts for a modification of the velocity field when the distance between the grooves is small ($a \rightarrow 1$). In our case, however, the simpler expression of single groove (3.2) is sufficiently accurate since the slip fraction is small (the difference between the expressions is smaller than 2 %).

[Figure 5\(b\)](#) compares the experimental results with the analytical solution (3.2). The measured values deviate from the elliptical profile predicted by the model; we observe instead a flat shape with a nearly constant local slip along the interface. To best of our knowledge, this constant spanwise slip velocity profile is not predicted by any existing model. One may speculate that it is due more complex Marangoni stresses than what is predicted by existing models (Landel *et al.* 2019; Sundin *et al.* 2021; Baier 2023).

Next, we compute the spanwise-average slip length measured at the interface, $\langle \beta^{int} \rangle$, and use it as a measure of the local interfacial slip. Note that the effective slip length, which is widely used to characterize surface slip, is related to the resistance the fluid experiences as it flows through the channel, and has thus a different physical interpretation than the average slip length. Moreover, for our experimental set-up and for non-flat interfaces, it is not straightforward to compute the effective slip (see more details in [Appendix F](#)).

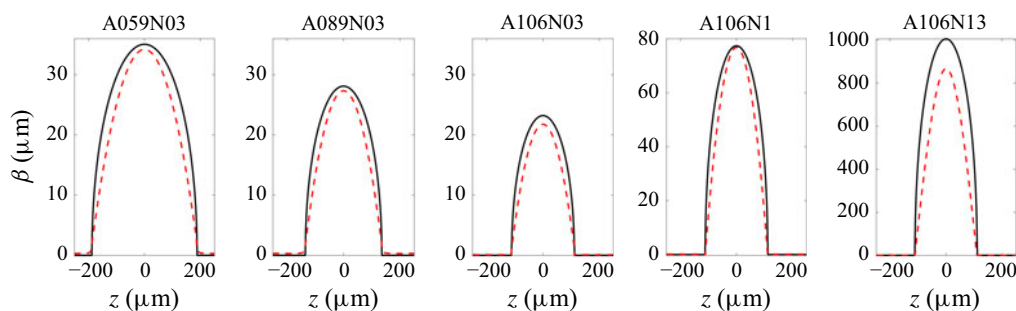


Figure 6. Effect of the curved interface on the local slip length. Continuous black lines correspond to $\beta^{Sch}(z)$, red dashed lines to the results of direct numerical simulations (DNS) (with meniscus curvature according to [table 1](#)).

The average slip length on the other hand is experimentally accessible and allows us to quantify the deviation of measured interfacial slip with predictions from theoretical and numerical simulations. We observe that $\langle \beta^{int} \rangle$, is significantly lower than the central value of the model distribution, $\beta^{Sch}(0)$. Specifically, as reported in [table 2](#), the ratio R between those two values varies from 5 to 7 for cases with small N and increases to 80 in the case with the highest viscosity ratio.

These observations may be explained by considering how our set-up deviates from the ideal model. The three most evident differences are: (i) the finite boundaries of the duct which limit the number and the length of the grooves; (ii) the curvature of the water–lubricant interface; (iii) the presence of the scattering medium in the water phase that can affect the dynamics of the interface.

Regarding the first point, we can deduce from the velocity measurements that in the central region of the duct (see [figure 4a](#)), the spanwise velocity variations are small, as the velocity field is nearly uniform in this region. On the other hand, the finite length of the grooves can cause a backflow of the lubricant opposing the water flow, contributing to a reduction of slip velocity with respect to the case of infinitely long grooves. The contribution of the shear stress arising from this backflow is discussed in the next section.

To investigate the impact of the meniscus at the liquid–liquid interface on the flow, numerical simulations were conducted. The computational set-up replicated the LIS designs and enforced the interface curvature observed in the experiments ([table 1](#)). A detailed description of the numerical set-up and methods is provided in §4. [Figure 6](#) shows the results obtained for each of the LIS configurations. The slip lengths obtained from simulations (dashed lines) at a curved interface are slightly smaller than the theoretical predictions of Schönecker & Hardt (2013). This indicates that the presence of a concave interface does not significantly modify the slip length distribution as observed in [figure 5\(a\)](#). The third point, i.e. the effect of contrast medium in the working fluid, is treated in the following section.

3.2. Surfactant contamination

As described in §2, to measure the flow velocity using D-OCT, the fluid must be light-scattering in the near-infrared. For this purpose, commercial milk (as previously described in §2.2) is mixed with water, as it provides a solid Doppler signal. However, milk might affect the interfacial properties between water and the lubricant infusing the LIS. Bovine milk contains two major proteins: casein and β -lactoglobulin, which account for

approximately 80 % and 20 % of the total protein content, respectively (O'Mahony & Fox 2014). Both these proteins have a hydrophilic and an oleophilic part to their structure, as detailed in McSweeney & Fox (2013). We may therefore regard them as soluble surfactants, which can accumulate at the water–lubricant interface, as well as being in solution. In a surfactant-laden fluid, the surfactant molecules adsorb at the interface (gas–liquid or liquid–liquid) and, advected by the flow, may accumulate at stagnation points (Manikantan & Squires 2020). This effect leads to a gradient of surfactant concentration which generates Marangoni stresses opposing the flow.

The observation that the slip velocity at the interface is significantly lower than predicted by the analytical model can be explained by taking into account the Marangoni stress into the tangential stress balance at the interface. At the water–lubricant interface the shear stress of the working fluid, τ_w , is balanced by two contributions. The first is given by the viscous resistance of the lubricant inside the groove, τ_l , while the second contribution is given by the Marangoni stress, τ_{Ma} . Thus, we can write

$$\tau_w = \tau_l + \tau_{Ma}. \quad (3.4)$$

The shear stress of the bulk fluid is calculated from the velocity measurements as $\tau_w = \mu_w \partial u / \partial y$ at the interface. The shear stress imposed by the lubricant on the interface can be estimated by assuming a velocity profile in the cavity. Neglecting interface curvature and confinement effects from the sidewalls of the duct, two limiting cases can be distinguished. In one case, we may assume that the lubricant is not conserved, i.e. lubricant is allowed to leave the grooves at the very downstream position, and if no new lubricant is entering upstream, the groove will eventually be drained (albeit very slowly). In this case, we expect a Couette profile in the groove and $\tau_l = \mu_l u_s / k$, where u_s is the slip velocity. In the second case, the lubricant is conserved because of a physical barrier downstream that creates a recirculating flow in the groove. We then have a pressure gradient that opposes the Couette flow to enforce lubricant conservation across the cavity section and $\tau_l = 4\mu_l u_s / k$ (Busse *et al.* 2013). We estimate the lubricant shear stress using the expression for the conserved lubricant, since the grooves at the very end of channel encounter a wall. The lubricant shear stress calculated with this hypothesis is reported in table 2.

An estimation of the Marangoni stress can be obtained by computing the ratio of τ_l to τ_w . A stress ratio near unity means that $\tau_{Ma} \ll \tau_l$, whereas a ratio near zero indicates $\tau_{Ma} \gg \tau_l$. As shown in table 2, where τ_l is estimated assuming lubricant conservation in the groove, Marangoni stresses dominate over the lubricant shear stress as $\tau_l / \tau_w \leq 0.3$ for all the configurations. In particular, we find that $\tau_{Ma} \approx 0.70\tau_w$ for the cases at low viscosity ratios and increases to $0.88\tau_w$ and $0.98\tau_w$ for $N = 1$ and $N = 13$, respectively.

A related configuration was treated by Sundin & Bagheri (2022), who modelled a two-dimensional flow carrying surfactants over transverse lubricated grooves. Their model is adapted to the present longitudinal configuration. The main difference lies in the fact that the Marangoni gradient builds along a much longer length (along $L = 8$ cm instead of the width of the groove $w \simeq 250$ μm). This calls for redefining some of the non-dimensional numbers of the problem from Sundin & Bagheri (2022). We derive the longitudinal model as detailed in Appendix C. The interpretation of the expressions is essentially the same for longitudinal and transverse configurations. We obtain the following expression for the effective slip length in the presence of surfactants in the bulk flow:

$$\beta_l^{Sun} = \beta_{SHS}^{0,l} b_{LIS}^l \left(1 - \frac{1}{1 + \alpha_d^l + \alpha_s^l} \right). \quad (3.5)$$

Quantity	Notation	Value
Bulk/surface diffusion constant	$D_s [\text{m}^2 \text{s}^{-2}]$	1.8×10^{-11}
Surfactant concentration in water	$c_0 [\text{mol m}^{-3}]$	0.03
Surfactant adsorption constant	$\kappa_a [\text{m}^3 \text{mol}^{-1} \text{s}^{-1}]$	2.3
Surfactant desorption constant	$\kappa_d [\text{s}^{-1}]$	1.4×10^{-5}
Ionic constant	n	2
Universal gas constant	$R [\text{J mol}^{-1}]$	8.314
Saturating surfactant concentration at interface	$\Gamma_m [\text{mol m}^{-2}]$	1.4×10^{-7}

Table 3. Physicochemical parameters of β -lactoglobulin used for the calculation of β_i^{Sun} (from Wahlgren & Elofsson (1997) and Rabe *et al.* (2007)). The concentration c_0 was calculated as a 20 % diluted solution of the standard concentration 0.3 g l^{-1} of β -lactoglobulin for commercial cow milk (Walstra, Wouters & Geurts (2005)).

Here, $\beta_{SHS}^{0,l} b_{LIS}^l$ is the effective slip length for a clean LIS. The surfactant diffusivity number

$$\alpha_d^l = \frac{1}{Pe_s Mac_k} \quad (3.6)$$

contains interfacial Péclet number

$$Pe_s = \frac{\hat{u}_{SHS}^{0,l} b_{LIS}^l w}{D_s} \quad (3.7)$$

where D_s (mol m^{-2}) is the surface diffusion coefficient of the surfactant and $\hat{u}_{SHS}^{0,l} b_{LIS}^l$ (m s^{-1}) is the estimated velocity at the interface of a clean LIS (depending only on the geometry and the viscosities of the working fluid and the lubricant). It also contains the Marangoni number

$$Ma = \frac{nRT\Gamma_m}{\mu_w U}, \quad (3.8)$$

which corresponds to the ratio of Marangoni forces ($nRT\Gamma_m$) to viscous forces $\mu_w U$. Here Γ_m (mol m^{-2}) is the saturating surface concentration of surfactants, while n , R , T are provided in table 3. We thus note the effects of surfactants on the effective slip is negligible if $\alpha_d^l \gg 1$, which corresponds to either a very small interfacial Péclet number (surface diffusion smears out all gradients) or a very small Marangoni number (Marangoni effect too small). Note that α_d^l also contains the non-dimensional bulk concentration of surfactants $c_k = \kappa_a c_0 / \kappa_d$, where κ_a ($\text{m}^3 \text{mol}^{-1} \text{s}^{-1}$), κ_d (s^{-1}) and c_0 (mol m^{-3}) are, respectively, the adsorption coefficient, desorption coefficient the reference bulk concentration of surfactants.

The interfacial solubility number

$$\alpha_s^l = \frac{1}{8} \frac{Bi'}{1 + Da_\delta} \frac{1}{Mac_k} \quad (3.9)$$

contains two additional non-dimensional numbers, namely, the Biot number

$$Bi' = \frac{\kappa_d L^2}{w \hat{u}_{SHS}^{0,l} b_{LIS}^l} \quad (3.10)$$

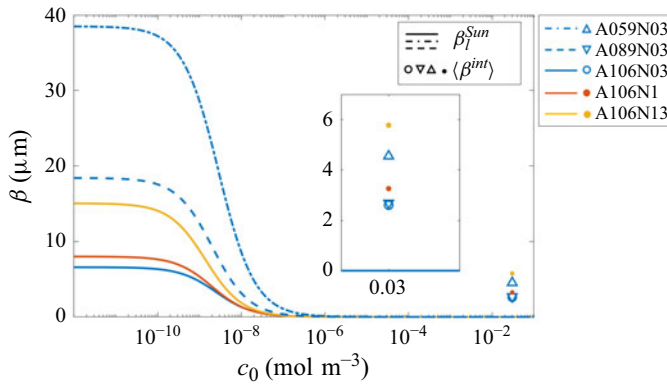


Figure 7. Effective slip lengths β_i^{Sun} predicted by Sundin's model as a function of the concentration c_0 in β -lactoglobuline (lines) and spanwise-averaged slip lengths measured at the interface $\langle\beta_i^{int}\rangle$ at the present estimated concentration (markers). The colours associated with the cases are: blue for A106N03, A106N1, A106N13; orange for A089N03; yellow for A059N03. Solid lines represent cases at $N = 0.3$, the dashed line represents $N = 1.2$ and the dash-dotted line $N = 13.2$.

and the Damköhler number

$$Da_\delta = \frac{\kappa_a \delta_L \Gamma_m}{D_s}, \quad (3.11)$$

where δ_L (m) is the typical length of depletion/excess in the bulk due to surface adsorption/desorption in the longitudinal case. Here, the effects of surfactants are small when $\alpha_s^l \gg 1$, which can be achieved for a very large Biot number (desorption kinetics dominates). The above explanations of α_s^l and α_d^l are simplified here, and more details of the different transport regimes can be found in Sundin *et al.* (2021).

The expression for $\beta_i^{Sun}(c_0)$ depends on the reference bulk concentration of surfactants c_0 through α_s^l and α_d^l , and allows us to calculate the expected value at the assumed concentration of β -lactoglobuline of our solution. The physicochemical parameters of β -lactoglobulin used to apply this model to the present case are provided by Wahlgren & Elofsson (1997) and Rabe *et al.* (2007) and reported in table 3. They were taken assuming the early times ($t < 3000$ s) kinetics from Rabe *et al.* (2007) (figure 7a in their article), where the adsorption/desorption rates are known and slower than the interfacial dynamics of the protein, making it behaving effectively as a simple poorly soluble surfactant.

Figure 7 shows β_i^{Sun} as a function of the bulk concentration of β -lactoglobulin (lines) compared with the spanwise-averaged slip length measured at the interface (symbols). For all our geometries, the model predicts $\beta_i^{Sun} \approx 0$ for bulk concentrations above $c_0 \sim 1 \times 10^{-6} \text{ mol m}^{-3}$. This threshold value is considerably lower than the estimated concentration of β -lactoglobulin for the working fluid, $c_{0,w} = 0.03 \text{ mol m}^{-3}$. The model thus indicates a total immobilization of the interface. In other words, the model predicts that the Marangoni stress at the interface completely dominates over the lubricant stress, i.e. $\tau_w = \tau_l + \tau_{Ma} \simeq \tau_{Ma}$.

This prediction of surfactant effects using Sundin's model and the simplified milk chemistry should be interpreted as a minimal estimate for interface immobilization, as other surfactants – such as casein, additional lactoglobulins or even potential contaminants from the channel walls – may also contribute. Including these components would likely lead to even lower critical concentrations of c_0 for the interface to remain mobile in figure 7. Our prediction therefore serves to highlight how far the system operates from

the mobile regime defined by the parameters in the Sundin and Landel models. It also does not imply that β -lactoglobulin alone is responsible for the observed immobilization, but rather that, if it were the only contributing factor, it would already be sufficient to account for the strong Marangoni gradients measured.

Note, however, that we measure non-zero slip lengths (figure 5*b*), which means that the interface retains more mobility than predicted by Sundin's model adapted to longitudinal grooves. It is not clear how the flow of surfactants at the interface behaves in this system and how it relates to adsorption and desorption dynamics. In fact, the experimental surfactant flow field (including surfactants at interfaces and in the bulk) can be more complex than what is assumed in the simplified geometry of the model (Baier 2023). A deeper understanding would require further investigations under controlled chemical conditions as well as accurate observation of the stagnation points. The model by Sundin & Bagheri (2022) nevertheless indicates that the reduced slip length, relative to the idealized LIS model, may result from surfactants in the system. To test this hypothesis, numerical simulations are performed, applying boundary conditions for either a clean mobile interface or a surfactant-laden immobile interface for the experimental configurations.

4. Numerical simulations

In this section, the numerical set-up and methodology are presented for modelling interface mobility. The obtained results are combined with the experimental measurements to better understand the effects of surfactants.

4.1. Configuration and method

We consider the two-phase configuration shown in figure 8. This domain represents a unit cell of the experimental configuration shown in figures 1 and 3. The two-dimensional domain is a square whose side (and therefore also the height) is the pitch p . Moreover, k is defined as in figure 1 and $l = p - k$ is the distance between the groove crest and the top boundary. Finally, θ is the angle that the local normal to the interface $\mathbf{n}$ forms with the vertical axis and ϕ is the deflection angle of the meniscus (also shown in figure 3). Periodic boundary conditions are applied in the spanwise direction, effectively simulating an infinite array of grooves. Moreover, no-slip boundary conditions are imposed on the solid surface. To drive the flow, a constant shear τ_∞ is applied at the top boundary. The variables are non-dimensionalized using the surface tension σ , fluid density ρ , and characteristic length scale l , matching the order of magnitude of the dimensionless groups from the experiments: $Re = \rho U l / \mu_w$, $We = \rho U^2 l / \sigma$ and $Ca = U \mu_w / \sigma$, where U is the velocity at $y = l$. Geometric properties (a , A) and the viscosity ratio N are also matched with the experiments. The two fluids, water and lubricant, have the same densities. Since the capillary and Weber numbers are small ($Ca \approx 10^{-4}$ and $We \approx 10^{-4}$), the interface is essentially static under the flow. It is initialized with a shape extracted from the OCT measurements (i.e. figure 3).

Numerical simulations were conducted using Basilisk (Popinet 2003, 2009, 2015), an open-source framework for solving partial differential equations on adaptive Cartesian meshes. The interface was tracked through a geometric volume-of-fluid (VOF) method (Scardovelli & Zaleski 1999). Despite the low Reynolds number, we solve the time-dependent equations, as it provides a more stable algorithm. The equations are discretized using a time-staggered approximate projection method on a Cartesian grid. The time discretization of the viscous diffusion term is treated via a second-order Crank–Nicolson fully implicit scheme, while spatial discretization employs second-order finite volume on an octree grid. The solid wall is modelled using an immersed boundary (cut-cell)

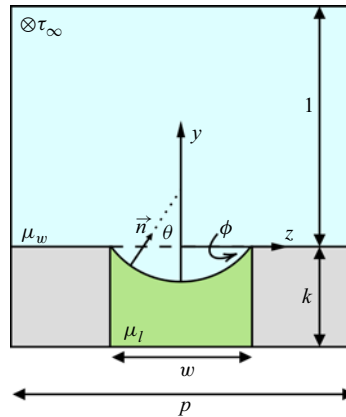


Figure 8. Schematic representation of the numerical set-up.

method (Johansen & Colella 1998; Mittal & Iaccarino 2005; Schwartz *et al.* 2006). The computational domain is a cubic volume with a side length equal to the pitch p of the investigated configuration, with periodic boundary conditions in the spanwise and streamwise directions, no-slip condition at the bottom boundary and the solid wall, and a Neumann condition at the top one. A two-dimensional slice of the domain is shown in figure 8 and represents the physical domain under consideration. The cubic computational domain was chosen to satisfy the constraints imposed by the VOF solver. Specifically, in this approach, the interface is reconstructed using a piecewise linear approximation and is advected based on the velocity at the cell faces. To accurately capture the interface dynamics, it is therefore essential to resolve the streamwise velocity components at both the front and back faces of the computational cells. Note that we impose a shear at the top boundary, rather than reproducing the full experimental channel geometry in the simulations. The latter is computationally too expensive.

The mesh refinement is based on a tree-grid discretization, with cell dimensions calculated as the domain length p divided by $2^{n_{ref}}$, where n_{ref} denotes the refinement level. The mesh refinement ranges from level 5 in the far field to level 8 near the interface and solid wall, ensuring that the cell size is at least 10 times smaller than the maximum local slip length. This refinement strategy is validated through a mesh independence study (see Appendix D).

The numerical method is validated by simulating the idealized case of a longitudinal groove with a flat interface, as considered in the models of Schönecker & Hardt (2013) and Schönecker *et al.* (2014). The effective slip lengths obtained numerically are compared with the analytical results and reported in Appendix D.

4.2. Comparison between experiments and simulations

To quantify the interfacial mobility of the experiments, we make use the numerical simulations to model the experimental configurations, which provides the full velocity fields above and within the cavity. The boundary conditions at the liquid–liquid interface in the simulations are tuned to represent two scenarios: (i) immobilized surfactant-laden interfaces and (ii) surfactant-free interfaces. In scenario (i), the immobilized regime is modelled by imposing zero velocity at the curved liquid–liquid interface. In scenario (ii), the continuity of velocity and tangential stress at the interface is enforced implicitly through the VOF to allow for local slip.

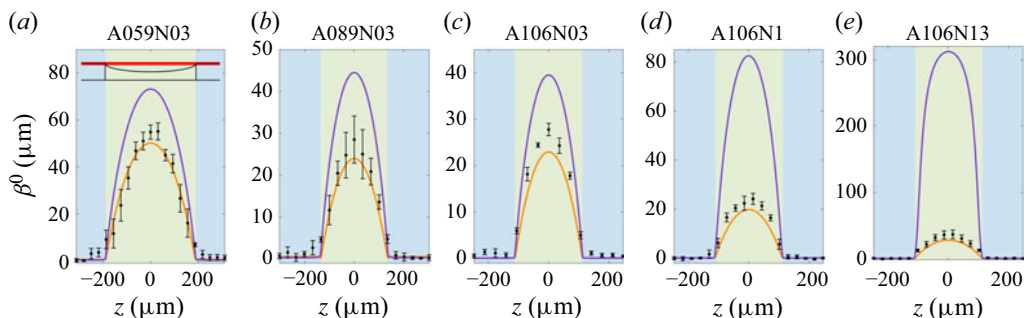


Figure 9. Experimental slip lengths $\beta^0(z)$ and the DNS results at the reference plane ($x, y = 0$) with no slip (orange line) and slip (purple line) boundary condition at the interface. The sketch in (a) represents the reference plane.

From the velocity fields, the local slip length distribution is computed by taking the ratio between the local normal shear rate and the velocity along a reference plane (as detailed in [Appendix F](#)). Since case (i) has, by definition, zero velocity at the liquid–liquid interface, we choose the reference plane to coincide with the crest plane $y = 0$ (shown in [figure 9a](#)). The slip lengths obtained from a simulation in scenarios (i) and (ii) are, respectively, denoted by $\beta_{NS}^0(z)$ and $\beta_S^0(z)$, where the subscripts ‘NS’ and ‘S’ refer to ‘no-slip’ and ‘slip’. The local slip length distributions from the simulations are compared with the experimental slip length obtained from (3.1). Note that experimental slip lengths are also evaluated at reference plane $y = 0$. The experimental slip length evaluated there is denoted by $\beta^0(z)$.

[Figure 9](#) compares the slip lengths extracted from the simulations with those from the experiments. The five frames correspond to the five experimental configurations listed in [table 1](#). The experimental results are represented with black symbols, while the simulated results are shown with solid lines (orange for case (i) and purple for case (ii)). For all cases, the experimental values $\beta^0(z)$ lie between the two predictions, but much closer to the immobilized case (i). The difference between maximum values of $\beta^0(z)$ and $\beta_{NS}^0(z)$ is smaller than 20 % for all cases, despite the simplifications inherent in the numerical model. In contrast, the slip lengths simulated with a fully mobile interface, $\langle \beta_S^0 \rangle$ are much larger than the experimental values. Furthermore, the difference between the numerical distributions with slip and no-slip conditions is significantly larger at higher viscosity ratios (cases A106N1 and A106N13). This is expected, given that the slip length in the absence of surfactants increases with N (see [figure 11](#) of [Appendix D](#)).

To gain further insight into how geometrical features affect the slip length, the average slip length is calculated and related to the aspect ratio of the grooves. The average slip length is defined as the arithmetic mean of the local slip length over a pitch (see [Appendices E](#) and [F](#)), i.e.

$$\langle \beta \rangle = \frac{1}{n} \sum_n \beta(z_n) \quad \text{with} \quad -\frac{p}{2} < |z_n| < \frac{p}{2}. \quad (4.1)$$

[Table 4](#) reports the average slip lengths of the LIS computed over the pitch (that is taking into account both the liquid–liquid and liquid–solid interfaces) and we observe again a good match between the experimental values $\langle \beta^0 \rangle$ and $\langle \beta_{NS}^0 \rangle$.

At fixed groove geometry, increasing the viscosity ratio (more viscous working fluid and/or less viscous lubricant) tends to get the system closer to the immobilized case (i). Increasing N by nearly a factor 50 (N goes from 0.3 to 13) results in a 30 % increase of the

Case	$\langle\beta^0\rangle$ [μm]	$\langle\beta_{NS}^0\rangle$ [μm]	$\langle\beta_S^0\rangle$ [μm]	τ_{Ma}/τ_w
A059N03	21.42 ± 1.33	20.82	31.19	0.91
A089N03	8.70 ± 0.19	7.49	14.17	0.84
A106N03	8.57 ± 1.63	7.04	12.44	0.69
A106N1	7.56 ± 0.71	6.09	26.22	0.93
A106N13	11.34 ± 1.89	8.51	106.36	0.97

Table 4. Average slip lengths derived from experiments $\langle\beta^0\rangle$ compared with the numerical results in case of immobile $\langle\beta_{NS}^0\rangle$ and slipping $\langle\beta_S^0\rangle$ interface.

average slip length in the presence of surfactants, whereas in the absence of contaminants, the increase is 88 %.

4.3. Discussion

For case (i), which models a fully immobilized interface, Marangoni stress dominates $\tau_{Ma} \approx \tau_w$, whereas for the surfactant-free case (ii), $\tau_{Ma} = 0$. These two limiting scenarios can be used to estimate the Marangoni stresses present in the experiments using the model by Sundin & Bagheri (2022). In their model, the effective slip is linearly dependent on the Marangoni stress. Assuming that this is also valid for the average slip length, we may fit a linear function using the two limiting values and estimate the stress ratio τ_{Ma}/τ_w given measurements of the average slip length.

The last column of table 4 reports the stress ratio for the five experimental cases. It can be observed that, the stress ratio τ_{Ma}/τ_w ranges between 0.7 to unity. From the tangential stress balance at the interface,

$$\frac{\tau_{Ma}}{\tau_w} = 1 - \frac{\tau_l}{\tau_w}. \quad (4.2)$$

The estimates confirm that for all configurations, the Marangoni stress is the main source of resistance at the interface, i.e. it dominates over the resistance imposed by the lubricant stress. The ratio between the stress exerted by the water and that exerted by the lubricant is proportional to the viscosity ratio, i.e. $\tau_l/\tau_w \propto \mu_l/\mu_w = 1/N$. Therefore from (4.2), we expect a larger Marangoni stress with increasing N , which is also confirmed in table 4. Note that in § 3.2 we estimated the Marangoni stress by estimating the lubricant shear stress as $\tau_l \approx 4\mu_l u_s/k$. The estimates using this approach (last column in table 2) are in good agreement with the estimate made here through a calibration with limiting values of average slip length obtained from numerical simulations (last column in table 4).

It is important to discuss the measured average slip length $\langle\beta^0\rangle$ in table 4, which, despite the minimum mobility of the liquid–liquid interface, amounts to 10 to 20 μm . The reported values represent the average slip at the crest plane of the ridges, i.e. the slip velocity is evaluated in the water phase rather than at the interface of the two fluids. This approach was previously used by Crowdy (2017) who treated a mathematically similar problem to ours. Crowdy studied a laminar flow over longitudinal grooves where the interface is immobile and the interface has a meniscus deflected by an angle ϕ from the plane $y = 0$. To derive the explicit expression of the effective slip length, Crowdy used a conformal mapping method to map the curved interface into the plane of the ridges. The simulated slip lengths of a completely immobile interface are in good agreement with Crowdy's predictions (see figure 13a in Appendix D). Another analytical approach accounting for the presence of surfactant is the model of Baier (2023), where the curved surfactant-laden interfaces are not immobilized but rather incompressible. In this case, the velocity profile

at the interface is non-zero with a zero average across the groove, thus predicting the presence of interfacial back-flows. This model explores other ways to add surfactants as a physical ingredient in LIS in a more refined way than immobilizing the interface. The average slip-length prediction remains the same as in the immobilized case of Crowdy (2017) (that is due to the presence of the meniscus). The predicted recirculations are not observed in our experiments.

5. Conclusions

For the first time, the local slip length over LISs with longitudinal grooves exposed to laminar flow has been measured through direct local velocity profiling. Using D-OCT, the shape of the liquid–liquid interface and the local velocity were measured with a micrometre spatial resolution. To investigate the role of the groove width and viscosity ratio on slip, five LISs configurations were constructed. The comparison between the measured slip lengths and the predictions of established analytical models (Schönecker & Hardt 2013), provided the starting point to quantify the influence of curved liquid–liquid interfaces and the presence of surfactants dispersed in the bulk flow.

The experimental cases were compared with numerical simulations where the boundary conditions were tuned to create two limiting scenarios, one of a surfactant-saturated interface and one of a surfactant-free interface. The first scenario corresponds to a rigid (no-slip velocity) corrugated surface, while the second scenario is a surface with finite local slip. Local slip length distributions over a pitch were calculated and compared with the corresponding experimental values. We measured nearly the same slip lengths experimentally as obtained from simulations of a rigid curved interface, indicating that we have strong resistance from Marangoni stresses, which immobilizes the interface. As explained by Sundin & Bagheri (2022), this stress is due to a gradient in surfactant concentration. The concentration is higher downstream than upstream in the groove resulting in a Marangoni stress that increases the resistance of the interface and thus reduces the slip velocity. The evidence of contaminants causing this detrimental effect is reinforced by the observation that, for a given groove geometry, the average slip length at $y = 0$ remains nearly constant and independent of the viscosity ratio. This suggests that the interface becomes more rigid, preventing the lubricating fluid from influencing the upper fluid as effectively.

However, although we have indications of interface immobilization, in our experiments we measured a small slip, which was larger than the experimental uncertainty. The non-zero slip lengths may be caused by a steady leakage of lubricant at the outlet of the grooves. The 8 cm-long longitudinal grooves used in this study are not closed at the edges. One may expect that by relaxing the stagnation points, there can be a small, but measurable, outgoing interfacial flux of surfactant downstream. This can lead to partially hindering the building of a Marangoni gradient and thus allowing for interfacial mobility despite the presence of surfactants. This leakage could be the enabling mechanism of our non-zero interfacial velocity measurements. Further investigations need to be carried out by measuring directly the flow field in the lubricant. This constitutes a future experimental challenge as a well-characterized contrast agent in the lubricant remains to be developed.

Our study indicates that a careful design of a LIS for a given working fluid and application is necessary to limit the effect of contaminants. Further studies are required to understand how surface texture and lubricant can be tuned to counteract, or at least minimize, the Marangoni stress. The presence of surfactant is not the main issue, it is the concentration gradient that needs to be counteracted, which may be achieved through the

design of geometry under given flow conditions. For example, the formation of surfactant concentration gradients can be modified by changing the stagnation points (e.g. barriers in the groove that stagnate the lubricant flow) of a LIS geometry. One of the most detrimental aspects of surface-active contaminants, is that they can result in failure of LISs in drag reduction applications. Further studies of interface mobility for different geometries (including random textures) are needed to determine the ubiquity of this failure mechanism. Our work, however, demonstrates the need for drag reduction studies to be cautious in their measurements of effective slip (and thus drag reduction) via global pressure drop and flow rate measurements. For example, if the height of the reference channel (channel with a Navier slip boundary condition imposed) corresponds to the distance between the crest plane of the grooved walls and the lubricant–liquid interface is deflected into the grooves, significant drag reduction can be measured despite a fully immobilized interface. Therefore, and as discussed in (Schönecker & Hardt 2015), drag reduction critically depends on the choice of the reference channel, and the appropriate choice may not be straightforward for certain flow configurations.

An additional contribution of our study is the first use of D-OCT as a relatively accurate technique to directly measure the slip velocity on a LIS and monitor the lubricant coverage at the same time. This technique achieves spatial resolution down to $2 \times 2 \times 2 \text{ } \mu\text{m}$ and a velocimetry precision around $10 \text{ } \mu\text{m s}^{-1}$ (see Amini *et al.* 2025). It is therefore an attractive technique to image even slightly mobile interfaces, and thus fill the gap between local measurements of slippage and global measurements of drag reduction. However, the requirement of an opaque fluid to perform the measurements limits the range of materials this method can be used with. For instance, D-OCT is well-suited for studying slip in blood flows or polymer suspensions, where the fluid's optical density eliminates the need for additional scattering agents. In LIS flows, this often comes (if not always) with an addition of potentially surface-active chemicals. Future work will use a contrast agent whose chemistry is more controlled, to determine the effect of exogenous/endogenous surfactants, as well as, to more quantitatively determine their influence on slippage. Eventually, there is room for improvement in the LIS preparation process (e.g. controlling the channel filling process better) that is likely to increase precision in statistics, albeit the interface mobilities reported herein would remain unaffected.

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Declaration of interests. The authors report no conflict of interest.

Appendix A. The LIS assembly and groove fabrication

A.1. Assembly of flow cell

As illustrated in figure 10, the flow cell consists of four square panels sandwiched between two aluminium plates, which are fastened by screws to ensure uniform pressure across the entire structure. The panels are arranged from top to bottom of the channel (left to right in figure 10) as follows: a transparent tile of polymethyl methacrylate (PMMA), a clear plastic sheet where the ridges are cast, a stainless steel tile with a rectangular slit of dimensions $3.25 \times 80 \text{ mm}^2$ and 1 mm-thick forms the duct, and a smooth hydrophilic

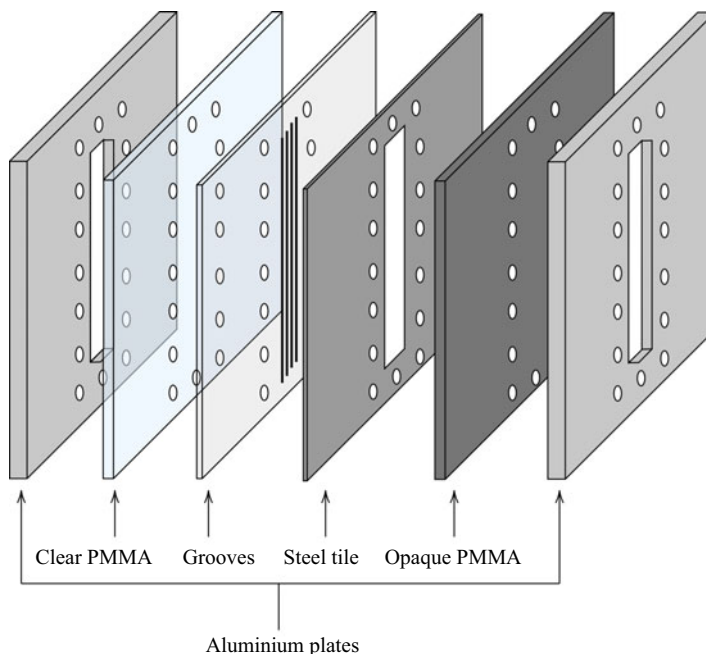


Figure 10. Schematic of the assembly of the flow cell.

tile made of opaque PMMA as the bottom of the duct. Mounting the grooves on the top wall of the duct is crucial for the accurate measurement of the velocity at the liquid–liquid interface. In this way, any spurious velocity measurement that can arise from the reflection of the OCT beam at the bottom wall is avoided.

A.2. Surface fabrication

The substrates of the LISs are fabricated from Ostemer 322 (Mercene Labs, Stockholm, Sweden), Off-Stoichiometry-Thiol-Ene resin via positive soft moulding with a silicon master made by deep ion etching. Components A and B of the resin are mixed with a speedmixer in ratio 1 : 1.09 as prescribed by the manufacturer and the mixture is poured on the mould. A transparent release liner applied on top facilitates the delamination from the mould and acts as a support layer to the surface. The sample is then exposed to an uncoherent UV-light source for 60 s and the solidified resin is removed from the mould. Finally, the sample is heat-cured at 100 °C for 1 hr. Subsequently, a hydrophobic spray coating (HYDROBEAD-T) is applied to the grooved surface, to ensure that when the surface is immersed, the lubricant wets the surface better than water. In this way, the established design requirements for LISs are satisfied (Wong *et al.* 2011).

In each experiment, the LIS was prepared through complete infusion of the duct with lubricant, followed by injection of the working fluid into the flow cell using a syringe pump (AL-1000, WPI) at a volumetric flow rate of 20 $\mu\text{L min}^{-1}$ until filled. The preparation procedure was continuously monitored via cross-sectional scanning of the duct with OCT. With this procedure, the lubricant is confined within the grooves for the full duct length.

With the chosen geometry and flow conditions, the lubricant layer will not experience shear-driven drainage. This is supported by estimating the maximum length of lubricant retained in the groove under shear flow, L_{∞} (Wexler *et al.* 2015). The retention length is

determined by the balance between shear force and capillary forces,

$$L_{\infty} = \frac{c_p}{c_s} \frac{k}{r_{min}} \frac{\sigma}{\tau_w}. \quad (\text{A1})$$

Here, $c_p/c_s \approx 1$ represents the ratio of non-dimensional geometric factors, r_{min} is the minimum radius of curvature of the interface, σ is the interfacial tension and τ_w is the shear stress (from the overlying flow). In our case, $r_{min} \approx 400 \mu\text{m}$, $\sigma \approx 50 \text{ mN m}^{-1}$ and $\tau_w \approx 4 \text{ mPa}$, resulting in an estimated retention length of $L_{\infty} \approx 4 \text{ m}$, far exceeding the length of the flow cell.

Appendix B. Uncertainty analysis

The uncertainty of the single slip length is calculated by propagating the errors associated with the velocity measurements. Considering (3.1), the overall uncertainty is estimated as follows:

$$\sigma_{\beta}^2 = \left(\frac{\partial \beta}{\partial u_s} \right)^2 \sigma_{u_s}^2 + \left(\frac{\partial \beta}{\partial \Delta} \right)^2 \sigma_{\Delta}^2, \quad (\text{B1})$$

where $\Delta = \partial u / \partial y$ is the velocity gradient evaluated at the interface. The uncertainty in the experimental measurement of the slip velocity, σ_{u_s} , is calculated as the standard deviation of the 1000 repeated scans needed to perform a single velocity acquisition and amounts on average to 5 % of the measured value. The velocity gradient Δ , corresponds to the fitting parameter that linearly interpolates the acquired velocity data close to the interface. Consequently, its standard deviation corresponds to the uncertainty derived from the fitting operation, which is 15 % on average.

In each configuration, the measurement is repeated in five spanwise positions where the interface curvature was rather constant and then their averaged is computed. The error bars shown in figure 5(b) correspond to the standard deviations of these repeated measurements, indicating the range in which 68 % of all the measurements is contained. Since the standard deviation is consistently larger than the propagated errors, this measure is considered as the variation of the experimental conditions.

Appendix C. Derivation of Sundin's model for longitudinal grooves

The model of Sundin & Bagheri (2022) provides an estimation of the effective slip length in the case of a surfactant-laden transverse flow imposed through a constant shear stress τ_{∞} . It takes into account the adsorption/desorption dynamics between surfactant in the bulk and at the interface using a Frumkin isotherm (Manikantan & Squires 2020) and solves the coupled Stokes problem with the resulting solutal Marangoni stress at the liquid–liquid interface. We aim at solving the same problem in the longitudinal configuration in order to provide a prediction for the effective slip length β_l^{Sun} of the groove in the presence of soluble surfactants.

In the present paper, the flow is in the longitudinal direction of the grooved surface. The finite length L of the grooves has to be taken into account as this is the axis along which a Marangoni stress builds up under shear.

To account for this, let us recall the notation for the present configuration, as described in figure 3: $\hat{y}$ is the vertical direction normal to the LIS; $\hat{z}$ is the spanwise/transverse direction of the grooves; $\hat{x}$ is the streamwise/longitudinal direction. Moreover, we recall (table 1) that our grooves have width $w = [223, 271, 376] \mu\text{m}$, pitch $p = [495, 608, 621] \mu\text{m}$ and length $L = 8 \text{ cm}$.

C.1. Definitions and governing equations

In order to derive the effective slip length in the longitudinal case, β_l^{Sun} , let us recall the system of coupled equations determining the interfacial dynamics. The hats notation stands for dimensionalized scalar fields and coordinates,

$$\Delta \hat{u} = 0 \quad \text{for} \quad \hat{y} > 0, \quad (C1)$$

$$\hat{\tau}_w = \hat{\tau}_l + \hat{\tau}_{Ma} \quad \text{where} \quad \hat{\tau}_w = \mu_w \frac{\partial \hat{u}}{\partial \hat{y}} \Big|_{\hat{y}=0^+} \quad \text{and} \quad \hat{\tau}_l = \mu_l \frac{\partial \hat{u}}{\partial \hat{y}} \Big|_{\hat{y}=0^-}, \quad (C2)$$

$$\hat{\tau}_{Ma} = -\frac{\partial \hat{\gamma}}{\partial \hat{x}}, \quad (C3)$$

$$\hat{\gamma} = \hat{\gamma}_0 - nRT\hat{\Gamma}, \quad (C4)$$

$$\frac{\partial(\hat{u}_s \hat{\Gamma})}{\partial \hat{x}} = \hat{S} + D_s \frac{\partial^2 \hat{\Gamma}}{\partial \hat{x}^2}, \quad (C5)$$

$$\hat{S} = \kappa_a \hat{c}_s (\Gamma_m - \hat{\Gamma}) - \kappa_d \hat{\Gamma}, \quad (C6)$$

$$-D \frac{\partial \hat{c}}{\partial \hat{y}} \Big|_{\hat{y}=0} = \hat{S}. \quad (C7)$$

The above equations represent, respectively, the momentum equation of the fluid above the interface, the tangential stress balance at the interface, the Marangoni stress, the linearized equation of state of the interface, the transport equation for the surfactants at the interface, the adsorption/desorption kinetics and finally, the balance between diffusive flux of surfactants and adsorption/desorption at the interface.

Let us also define the mean spanwise interfacial velocity

$$\hat{u}_s^{0,l} = \frac{1}{p} \int_{-p/2}^{p/2} \hat{u}_s d\hat{z}, \quad (C8)$$

and the effective slip length of the longitudinal groove

$$\beta_l^{Sun} = \frac{\mu_w}{\tau_w} \frac{1}{L} \int_{-L/2}^{L/2} \hat{u}_s^{0,l} d\hat{x}. \quad (C9)$$

C.2. Assumptions

The main assumptions are the following.

- (i) The surface and bulk diffusion coefficients of the surfactants are equal, $D = D_s$.
- (ii) The interfacial concentration of surfactants $\hat{\Gamma}_0$ that is at equilibrium with the bulk concentration $\hat{c}_0$ is low compared with the saturating concentration of the interface, $\hat{\Gamma}_0 \ll \Gamma_m$.
- (iii) An important hypothesis of the model of Sundin & Bagheri (2022) (also enforced here) is $\hat{\tau}_{Ma} = \text{constant}$. This means that we only account for linear profiles of surface tension $\hat{\gamma}$ and surface concentration $\hat{\Gamma}$ along $\hat{x}$, and invariant along $\hat{z}$. The same is assumed for the bulk concentration of surfactants at the interface $\hat{c}_s = \hat{c}(\hat{y} = 0)$.

Besides linearizing, we also assume that the equilibrium values $(\hat{\gamma}_0, \hat{r}_0, \hat{c}_0)$ are reached in the middle of the groove at $\hat{x} = 0$, leading to

$$\hat{\gamma} = \hat{\gamma}_0 - \frac{2\hat{x}}{L} \Delta \hat{\gamma}, \quad (\text{C10})$$

$$\hat{r} = \hat{r}_0 + \frac{2\hat{x}}{L} \Delta \hat{r}, \quad (\text{C11})$$

$$\hat{c}_s = \hat{c}_0 + \frac{2\hat{x}}{L} \Delta \hat{c}_s. \quad (\text{C12})$$

- (iv) The vertical variations of the bulk concentration of surfactants are assumed to vary linearly along a typical length δ_L so that

$$\frac{\partial \hat{c}}{\partial \hat{y}}(\hat{x}) = \frac{c_0 - \hat{c}_s(\hat{x})}{\delta_L} = -\frac{2\hat{x}}{L} \frac{\Delta \hat{c}_s}{\delta_L}. \quad (\text{C13})$$

The quantity δ_L can be estimated for high Péclet number (which will be our case here) from the bulk transport equation of surfactant,

$$\hat{u} \frac{\partial \hat{c}}{\partial \hat{x}} = D \frac{\partial^2 \hat{c}}{\partial \hat{y}^2}. \quad (\text{C14})$$

We integrate both sides of it along $\hat{x} \in [-L/2, 0]$ and $\hat{y} \in [0, +\infty]$. The profile of concentration in the bulk at any abscissa $\hat{x}$ is assumed to vary linearly from $\hat{c}_s(\hat{x})$ to c_0 when $\hat{y} \in [0, \delta_L]$ and to plateau at c_0 when $\hat{y} \in [\delta_L, +\infty]$ so that it can be expressed as

$$\hat{c}(\hat{x}, \hat{y}) = \begin{cases} c_0 + \frac{2\hat{x}\Delta\hat{c}_s}{L} \frac{\delta_L - \hat{y}}{\delta_L} & \text{if } \hat{y} \in [0, \delta_L], \\ c_0 & \text{if } \hat{y} \in [\delta_L, +\infty]. \end{cases} \quad (\text{C15})$$

We also approximate the velocity field to be in the L  v  que regime as in Landel *et al.* (2019),

$$\hat{u}(\hat{x}, \hat{y}) \simeq \frac{\tau_\infty}{\mu_w} \hat{y}. \quad (\text{C16})$$

Integrating (C14) and inserting (C15)–(C16), the left-hand side becomes

$$\int_0^{+\infty} \int_{-L/2}^0 \hat{u} \frac{\partial \hat{c}}{\partial \hat{x}} d\hat{y} d\hat{x} = -\frac{\tau_\infty \Delta \hat{c}_s}{\mu_w} \frac{\delta_L^2}{6}, \quad (\text{C17})$$

and the right-hand side becomes

$$\int_0^{+\infty} \int_{-L/2}^0 D \frac{\partial^2 \hat{c}}{\partial \hat{y}^2} d\hat{y} d\hat{x} = -\frac{DL \Delta \hat{c}_s}{4\delta_L}, \quad (\text{C18})$$

leading to the scaling

$$\delta_L = \left(\frac{3}{2} \frac{wL^2}{Pe_L} \right)^{1/3}. \quad (\text{C19})$$

Here, $Pe_L = UL/D$ is the bulk P  clet number (defined in table 6). A typical value computed for the experiment A106N03 yields $\delta_L = 60 \mu\text{m}$. Note that the L  v  que regime is valid as long as $\delta_L \gg w\hat{u}_s/U$ which can be estimated *a posteriori* as $\simeq 0.05w \simeq 10 \mu\text{m}$.

- (v) The velocity field $\hat{u}_s$ at the interface is assumed to depend only on $\hat{z}$ and the solution of (C1) in the longitudinal case is given by (Schönecker *et al.* 2014)

$$\hat{u}_s = \hat{u}_s^{Sch,l}(\hat{z}) = \frac{p(\hat{\tau}_\infty - \hat{\tau}_w)}{\pi \mu_w} \text{Arccosh} \left(\frac{\cos(\alpha \hat{z}/w)}{\cos \alpha} \right) \quad (\text{C20})$$

Here, $\alpha = \pi w/(2p)$ is defined by the transverse geometry of the grooves. Equation (C8) leads to the average interfacial flow

$$\hat{u}_s^{0,l} = \frac{1}{p} \int_{-p/2}^{p/2} \hat{u}_s^{Sch,l}(\hat{z}) d\hat{z} = -\frac{p(\hat{\tau}_\infty - \hat{\tau}_w)}{\pi \mu_w} \ln(\cos \alpha). \quad (\text{C21})$$

- (vi) The viscous shear stress in the surfactant-free lubricant is assumed to be linked to the mean interfacial velocity through the same relationship as in the clean case given by (Schönecker *et al.* 2014)

$$\hat{u}_s^{0,l} = -\frac{P}{\pi \mu_w} C_l \hat{\tau}_l \ln(\cos \alpha), \quad (\text{C22})$$

where C_l is a factor that accounts for the fraction of momentum flux transferred from the shear-driven flow $\hat{\tau}_\infty$ to the shear at the interface $\hat{\tau}_w$. In the case of a clean interface (i.e. $\hat{\tau}_{Ma} = 0$), $\hat{\tau}_w = \hat{\tau}_\infty/(1 + C_l)$. It is derived by Schönecker *et al.* (2014) and reads

$$C_l = \frac{4\alpha D_l N}{\ln \left(\frac{1+\sin(\alpha)}{1-\sin(\alpha)} \right)}, \quad (\text{C23})$$

with $D_l(A, a) \simeq 0.33 - 0.34$ for the cases $A = 0.59 - 1.06$; it depends only on the groove geometry and varies weakly when $A > 0.5$, and $N = \mu_w/\mu_l$ is the viscosity ratio.

Rewriting (C21) using (C22) and (C2), we have

$$\hat{u}_s^{0,l} = \hat{u}_{SHS}^{0,l} b_{LIS}^l \left(1 - \frac{\hat{\tau}_{Ma}}{\hat{\tau}_\infty} \right), \quad (\text{C24})$$

$$\text{where } \hat{u}_{SHS}^{0,l} = -\frac{p}{\pi} \frac{\tau_\infty}{\mu_w} \ln(\cos \alpha), \quad (\text{C25})$$

$$\text{and } b_{LIS}^l = \frac{C_l}{1 + C_l}, \quad (\text{C26})$$

where $\hat{u}_{SHS}^{0,l}$ is the spanwise mean velocity at the interface for a longitudinal flow over a clean SHS of the same geometry.

C.3. Non-dimensionalization

Equations (C1) to (C7) are non-dimensionalized with: $x = \hat{x}/w \in [-L/2w, L/2w]$, $y = \hat{y}/w \in [0, +\infty[$, $z = \hat{z}/w \in [-1/2, 1/2]$, $U = w\tau_\infty/\mu_w$, $c = \hat{c}/c_0$, $\Gamma = \hat{\Gamma}/\Gamma_m$, and we introduce the quantities and numbers defined, respectively, in tables 5 and 6. Note that κ_d/κ_a is homogeneous to a concentration, called the Langmuir concentration, which is the equilibrium bulk concentration at the interface (i.e. when the source $\mathcal{S}$ is zero) when $\Gamma = \Gamma_m/2$, half the saturating concentration Γ_m ,

Quantity	Non-dimensional expression
Velocity	$u = \hat{u}/U$
Viscosity ratio	$N = \mu_w/\mu_l$
Groove interface aspect ratio	$\varepsilon = w/L$
Shear stress of the working fluid at the interface	$\tau_w = \hat{\tau}_w/\tau_\infty = \frac{\partial u}{\partial y} \Big _{y=0^+}$
Shear stress of the lubricant at the interface	$\tau_l = \hat{\tau}_l/\tau_\infty = \frac{1}{N} \frac{\partial u}{\partial y} \Big _{y=0^-}$
Marangoni stress	$\tau_{Ma} = \hat{\tau}_{Ma}/\tau_\infty$
Surface tension	$\gamma = \hat{\gamma}/(\mu_w U)$
Interfacial surfactant concentration	$\Gamma = \hat{\Gamma}/\Gamma_m$
Bulk surfactant concentration	$c = \hat{c}/c_0$
Langmuir isotherm source term	$\mathcal{S} = w\hat{\mathcal{S}}/(U\Gamma_m)$
Bulk depletion length	$\delta_L = \hat{\delta}_L/w$

Table 5. Non-dimensional quantities used in the model of Sundin & Bagheri (2022) adapted to longitudinal flows.

Number	Expression	Interpretation
Bulk Péclet number	$Pe_L = \frac{UL}{D}$	Bulk advection/bulk diffusion
Surface Péclet number	$Pe_s = \frac{u_{SHS}^{0,l} b_{LIS}^l w}{D_s}$	Surface advection/surface diffusion
Biot number	$Bi = \frac{\kappa_d L}{U}$	Desorption flux/interfacial advection flux
Corrected Biot number	$Bi' = \frac{\kappa_d L^2}{wu_{SHS}^{0,l} b_{LIS}^l}$	Desorption flux/interfacial advection flux (corrected)
Damköhler number	$Da_\delta = \frac{\kappa_a \delta_L \Gamma_m}{D_s}$	Adsorption flux/interfacial diffusion
Marangoni number	$Ma = \frac{nRT\Gamma_m}{\mu_w U}$	Marangoni stress/viscous stress in the working fluid
Normalized bulk concentration	$c_k = \frac{\kappa_a c_0}{\kappa_d}$	Imposed bulk concentration/Langmuir concentration

Table 6. Non-dimensional numbers used in the model of Sundin & Bagheri (2022) adapted to longitudinal flows.

$$\Delta u = 0 \quad \text{for } y > 0, \quad (\text{C27})$$

$$\tau_w = \tau_l + \tau_{Ma}, \quad (\text{C28})$$

$$\tau_{Ma} = -\frac{\partial \gamma}{\partial x}, \quad (\text{C29})$$

$$\gamma = \gamma_0 - Ma\Gamma, \quad (\text{C30})$$

$$\frac{\partial(u_s \Gamma)}{\partial x} = \mathcal{S} + \frac{1}{\varepsilon Pe_L} \frac{\partial^2 \Gamma}{\partial \hat{x}^2}, \quad (\text{C31})$$

$$\mathcal{S} = \varepsilon Bi \left(c_s c_k (1 - \Gamma) - \Gamma \right), \quad (\text{C32})$$

$$-\varepsilon Bi \frac{\delta_L}{Da_\delta} c_k \frac{\partial c}{\partial y} \Big|_{y=0} = \mathcal{S}. \quad (\text{C33})$$

The linearized profiles of surface tension, interfacial concentration of surfactants can be rewritten as

$$\gamma = \gamma_0 - 2\varepsilon \Delta \gamma \, x, \quad (\text{C34})$$

$$\Gamma = \Gamma_0 + 2\varepsilon \Delta \Gamma \, x. \quad (\text{C35})$$

We can rewrite (C24) in the form

$$u_s^{0,l} = u_{SHS}^{0,l} b_{LIS}^l (1 - \tau_{Ma}) \quad (\text{C36})$$

with $u_{SHS}^{0,l} = -(1/2\alpha) \ln(\cos \alpha)$ the non-dimensional interfacial mean velocity for a clean SHS.

C.4. Expression for the effective slip length

The normalized effective slip length of the longitudinal groove simplifies to

$$\beta_l^{Sun}/w = u_s^{0,l} = u_{SHS}^{0,l} b_{LIS}^l (1 - \tau_{Ma}). \quad (\text{C37})$$

In order to predict the effective slip length, we now need to make the Marangoni stress explicit. Using (C29) and (C30), it writes as

$$\tau_{Ma} = 2Ma\varepsilon \Delta \Gamma \quad (\text{C38})$$

which can be substituted in (C37). On the other hand, the transport equation of surfactants at the interface (C31) reads, using $\Gamma_0 \ll 1$, as

$$u_s^{0,l} = \frac{2}{c_k} \left(\frac{1}{Pe_L} + \frac{1}{8} \frac{Bi}{1 + Da_\delta} \right) \Delta \Gamma. \quad (\text{C39})$$

This is an alternative form of (C37) that allows us to find the expression of $\Delta \Gamma$. Eventually, we put the effective slip length in the form

$$\beta_l^{Sun} = \beta_{SHS}^{0,l} b_{LIS}^l \left(1 - \frac{1}{1 + \alpha_d^l + \alpha_s^l} \right), \quad (\text{C40})$$

where

$$\beta_{SHS}^{0,l} = w u_{SHS}^{0,l}, \quad (\text{C41})$$

$$\alpha_d^l = \frac{1}{Mac_k Pe_s}, \quad (\text{C42})$$

$$\alpha_s^l = \frac{1}{8} \frac{Bi'}{1 + Da_\delta} \frac{1}{Mac_k}. \quad (\text{C43})$$

The relative values of α_d^l and α_s^l compare which mechanism dominates the evacuation of surfactants on a given piece of interface: if $\alpha_d^l \ll \alpha_s^l$, desorption towards the bulk is the main mechanism, if $\alpha_d^l \gg \alpha_s^l$, surface diffusion dominates. Note that both numbers scale as $\sim 1/Ma$, meaning that for Marangoni numbers which are large enough ($Ma \rightarrow \infty$, great capacity of the interface to create a Marangoni stress), they both become small and the interface becomes immobilized ($\beta_l^{Sun} \rightarrow 0$).

The expression (C40) of β_l^{Sun} is thus used to calculate the estimation of figure 7 using the values of physicochemical parameters of table 3, and the values of non-dimensional numbers of table 7.

Appendix D. Validation of the numerical set-up

The validation of the numerical simulations was performed by reproducing the configurations considered by Schönecker *et al.* (2014) and comparing the results with their

Variable	Notation	Value
Interfacial Péclet number	Pe_s	890
Marangoni number	Ma	254
Normalized bulk concentration	c_k	4.9×10^3
Biot number (corrected)	Bi'	0.015
Damköhler number	Da_δ	0.69
Interfacial diffusivity number	α'_q	8.3×10^{-10}
Surfactant solubility number	α'_s	1.0×10^{-4}

Table 7. Non-dimensional numbers used for the definition of the effective slip length β_l^{Sun} for a longitudinal LIS in the presence of surfactants. The reported values are calculated for case A106N03. These numbers are in the same order of magnitude for the other four cases.

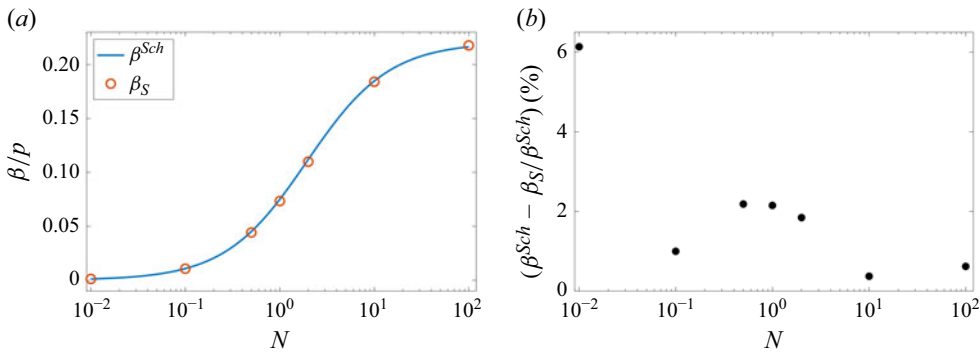


Figure 11. (a) Comparison between the normalized effective slip lengths β/p obtained from numerical simulations and the analytical model of Schönecker *et al.* (2014) as a function of the viscosity ratio N . (b) Relative error between the two evaluations. In these tests, $A = 0.5$ and $a = 0.667$.

analytical solutions. The validation set up is the same one described in § 4, with the angle $\phi = 0$, i.e. a flat water–lubricant interface.

In the first validation, a cavity with aspect ratio $A = 0.5$ and slipping area fraction $a = 0.667$ was used, and the viscosity ratio varied from $N = 0.01$ to $N = 100$ to assess the quality of the results for seven different cases. The effective slip length was calculated from the simulated velocity field as

$$\beta_S^Q = \frac{Q}{p \frac{\tau_\infty}{\mu_w} H} - \frac{H}{2} \quad (D1)$$

where Q is the volumetric flow rate obtained with τ_∞ applied at the top boundary, and compared with (2.29) in Schönecker *et al.* (2014). Figure 11(a) shows that the values of the numerical and analytical effective slip lengths normalized with the pitch, β/p , are in good agreement. The relative errors between the two evaluations, reported in figure 11(b), are below 3 % for all the tested cases, except at $N = 0.01$ where it increases to 6 %.

Similarly, the validation test is repeated with the dimensions of the cavity aligned with those of the groove dimensions employed in the experimental configurations, as well as the viscosity ratios ($N = 0.3, 1.2, 13.2$). Also in this case the interface between the two fluids is flat. We find again good agreement between the simulated effective slip lengths and the analytical predictions, as shown in figure 12(a), with a small relative error (figure 12b).

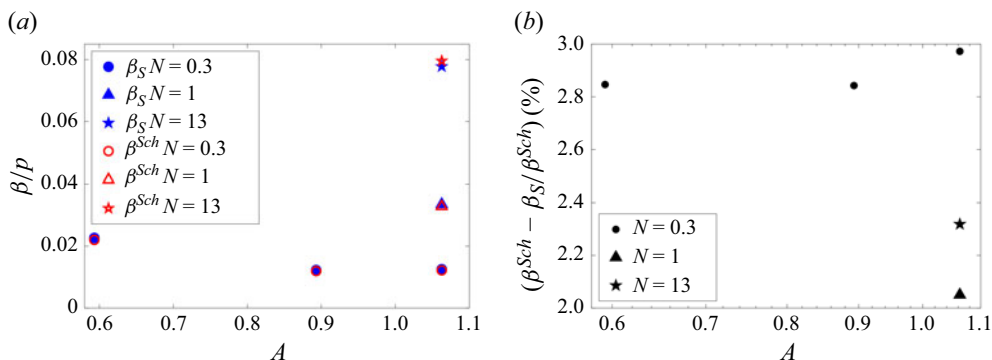


Figure 12. (a) Comparison between the normalized effective slip lengths β/p obtained from numerical simulations and the analytical model of Schönecker *et al.* (2014) as a function of the aspect ratio A . (b) Relative error between the two evaluations. The geometrical dimensions of the cavity and the viscosity ratios reproduce the five experimental cases and the interface is flat.

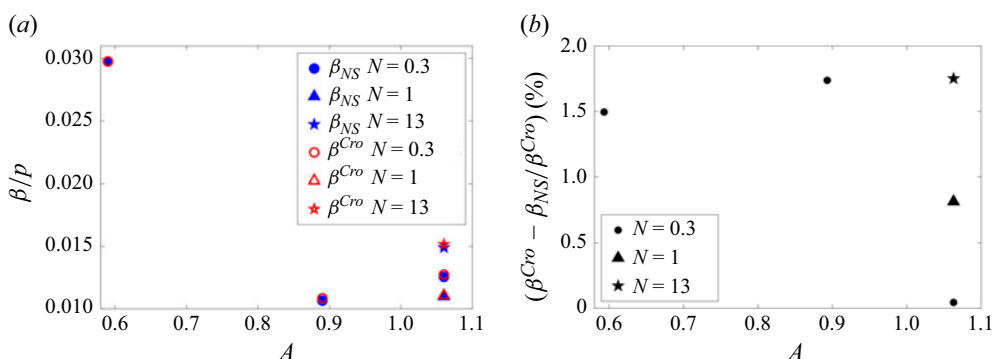


Figure 13. (a) Comparison between the normalized effective slip lengths β/p obtained from numerical simulations and the analytical model of Crowdy (2017) as a function of the aspect ratio A . (b) Relative error between the two evaluations. The geometrical dimensions of the cavity and the viscosity ratios reproduce the five experimental cases with a curved interface.

The simulated velocity field and the relative effective slip lengths in the scenario of an immobilized interface with a curved meniscus are validated through the comparison of the numerical results with the analytical derivation of Crowdy (2017). Equation (1.5) in Crowdy (2017) was used to calculate the predicted value of the effective slip length, β^{Cro} , where the geometrical features of the experiments (groove dimensions and meniscus deflection) have been matched. Good agreement between the effective slip lengths obtained from simulations and theory can be observed in figure 13(a) and the small relative errors are shown in figure 13(b).

The final assessment entails a comparison of the normalized local slip length distributions, β/p , obtained numerically and analytically, with the same configurations used in the experiments. The local slip length is calculated from the numerical velocity field as

$$\frac{\beta}{p} = \frac{u_s}{\frac{\partial u}{\partial y}|_{y=0}} \quad (D2)$$

where the slip velocity u_s is evaluated at $y = 0$.

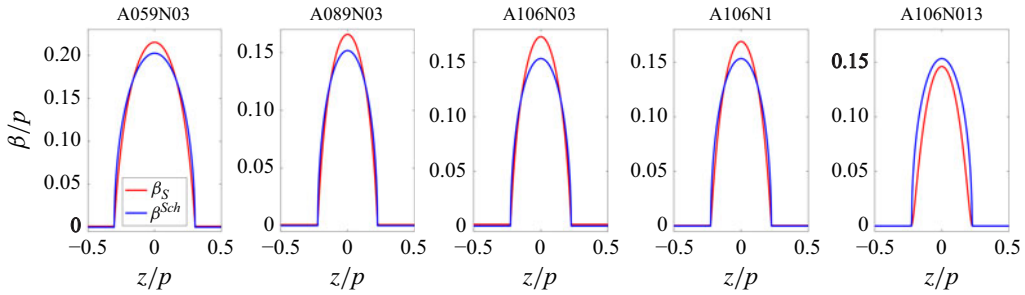


Figure 14. Local slip length comparison between the numerical result and the analytical model.

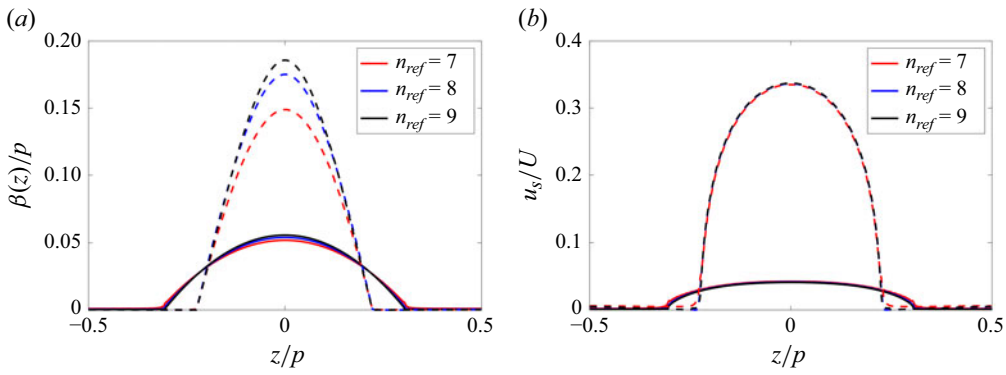


Figure 15. Mesh convergence study for three different levels of refinement. Two cases are shown, i.e. $A = 0.59$ with $N = 0.3$ (solid lines) and $A = 1.06$ with $N = 13$ (dashed lines). (a) Local slip length. (b) Local slip velocity.

The analytical expression used to calculate β^{Sch}/p as comparison is given by (2.28) in Schönecker *et al.* (2014). The results are shown in figure 14 where each of the five frames corresponds to a test case. The trend and magnitude of the predicted local slip lengths are reproduced by the numerical model with good agreement.

Regarding the refinement level used for the simulations, a mesh independence analysis was conducted, as illustrated in figure 15. The figure shows the local slip length (figure 15a) and slip velocity (figure 15b) for two distinct cases involving a meniscus, namely $A = 0.59$ with $N = 0.3$ (solid lines) and $A = 1.06$ with $N = 13$ (dashed lines) obtained with three different levels of refinement $n_{ref} = 7, 8, 9$ that correspond to a minimum cell size of, respectively, $p/2^7$, $p/2^8$ and $p/2^9$. While both the slip length and the slip velocity for $N = 0.3$ are practically independent of mesh refinement, it is observed that for $N = 13$, convergence is only achieved for the slip velocity, while the slip length is approaching convergence but is not yet fully mesh independent. This outcome is attributed to the observation that, for higher viscosity ratios, the velocity profile in proximity to the interface exhibits a less linear character compared with the behaviour observed for lower values of N . Consequently, while the velocity converges with reduced resolutions, the velocity gradient (utilized to calculate the slip length) necessitates higher refinement levels, resulting in a solution that is more contingent on the mesh size. Given the increase in computational cost associated with higher mesh refinement, it was decided to maintain a refinement level of 8, which is sufficient to ensure the mesh independence of the velocity field and the slip length for the majority of cases, yielding only minor errors for $N = 13$.

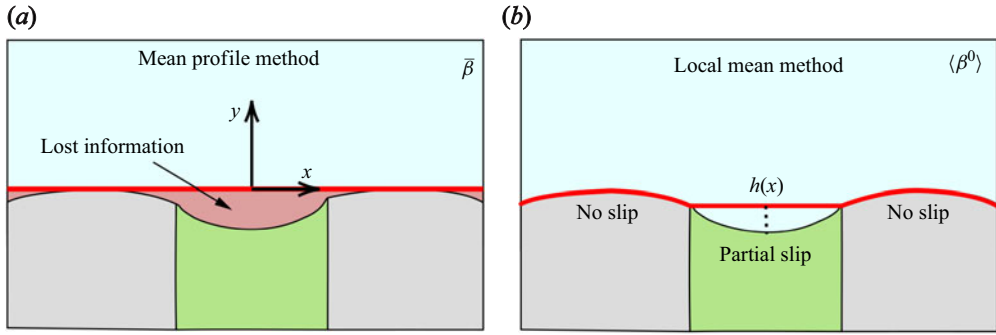


Figure 16. Illustration of two methods for the calculation of β_{eff} from measurements. (a) In the mean profile method, the horizontal plane tangent to the ridges' top is considered. (b) In the local mean method, the boundary is composed of the profile of the ridges and the height function $h(z)$ at the liquid interface. The top right-hand corners indicate the variable obtained with each method.

As these results were utilized in a comparison with experimental data, for which the slip values were considerably smaller than those computed from the simulations, these errors do not affect the results or the conclusions. It is also noteworthy that this convergence occurred at the interface, while the majority of results are presented for $y = 0$, where the mesh convergence is reached for lower refinement.

Appendix E. Experimental estimation of the average slip length

The experimental set-up used in the study does not allow to measure accurately the flow rate. Therefore average values of measurements are used to characterize the net effect of the LIS on the upper flow. In the following, we compare two approaches to calculate the average slip length and identify the most suitable for LISs with a non-flat, patterned substratum.

The first approach referred to as the mean profile method, has been employed in previous studies (Bolognesi *et al.* 2014; Schäffel *et al.* 2016). In this method, the velocity profiles are averaged over the span of a pitch, an operation that implies considering as a reference the plane tangent to the crest of the solid pattern, as sketched in figure 16(a). The resulting profile is then fitted to a function that describes a pressure-driven flow with a partially slippery wall,

$$\bar{u}_{\text{fit}}(y) = 4U_m \frac{(\beta_{\text{fit}} + H)(H - y)[\beta_{\text{fit}}(y + H) + yH]}{H^2(2\beta_{\text{fit}} + H)^2}, \quad (\text{E1})$$

with the maximum velocity of the mean profile, U_m , the channel height H and the effective slip length β_{fit} being the fitting parameters. An example of this analysis applied to the present data is shown in figure 17(a), where the experimental mean profile, $\bar{u}_{\text{exp}}(y)$, is fitted to (E1). This method performs well if the ridge of the solid structure is flat, and the no-slip velocity regions can be fully captured during the averaging process. However, in the case of a non-flat solid boundary, information loss can occur where the reference plane deviates from the actual structure, as illustrated in the sketch of figure 16(a). As a result, the slip velocity of the mean profile is overestimated, leading to an overestimation of the effective slip length. To quantify the accuracy of this method, we compared β_{fit} with the effective slip length obtained as $\bar{\beta} = \bar{u}_s / (\partial \bar{u} / \partial y)$.

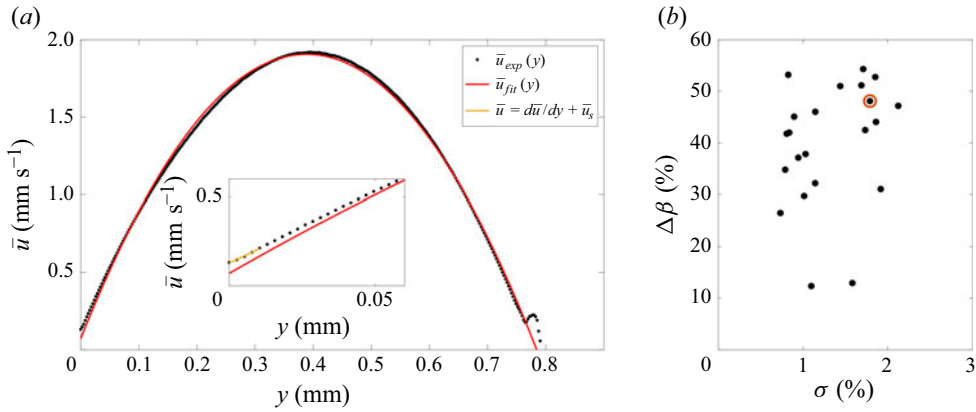


Figure 17. Sensitivity analysis for the mean profile method. (a) Mean velocity profile for case A106N13 fitted to EI (red line). The insert shows an enlargement of the near wall area where the linear fit of the mean profile is added. (b) The relative difference of the effective slip length calculated as a fitting parameter and according to Navier's definition with respect to the relative error of the fit. The red circle marks the case of (a).

The relative uncertainty of the fit is estimated as

$$\sigma = rms \left(\frac{u}{U_m}, \frac{\bar{u}_{fit}}{U_m} \right) \quad (\text{E2})$$

and the relative difference between the effective slip lengths is given by

$$\Delta\beta = \frac{\bar{\beta} - \beta_{fit}}{\bar{\beta}}. \quad (\text{E3})$$

As shown in figure 17(b), although the uncertainty of the fit is only 1 %, the relative difference $\Delta\beta$ ranges from 13 % in the best case to 54 % in the worst case, highlighting the sensitivity of this method to the averaged velocity.

The second approach, named local mean method, involves a reverse procedure compared with the previous one. In this case, the liquid–solid and liquid–liquid interfaces are treated separately, as depicted by the red profile in figure 16(b). The local slip length considered here is $\beta^0(z)$ described in § 4.2. The average slip length is then determined by averaging these local values along the streamwise and spanwise directions over the span of a pitch and it is denoted as $\langle\beta^0\rangle$.

Appendix F. Effective slip length versus average slip length

From the velocity field computed in the numerical simulations, the volumetric flow rate of the working fluid Q can be calculated. Using (D1), this allows us to extract the effective slip length, denoted β_S^Q for the surfactant-free case and β_{NS}^Q for the immobilized interface. These definitions are global: they capture the integrated effect of the boundary conditions on the overall flow, including both local slip and geometric effects such as changes in cross-section due to the meniscus shape. It is often the quantity of interest when modelling or predicting the drag-reduction of patterned surfaces such as LIS or SHS.

In contrast, we also compute spatially averaged slip lengths, derived directly from the local velocity field using Navier's condition (i.e. the ratio of tangential velocity to shear rate), either along the reference plane $y = 0$ or along the actual fluid–fluid interface. In the plane $y = 0$, we define and compute with DNS $\langle\beta_S^0\rangle$ for a clean slippery interface,

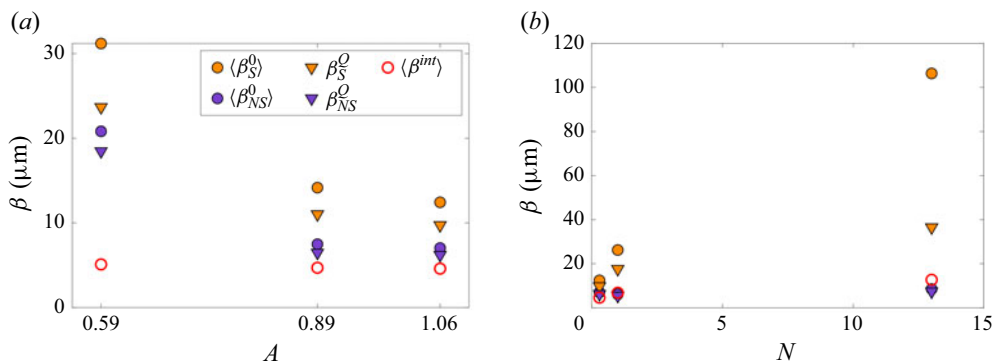


Figure 18. Effective slip length β^Q , average slip length $\langle \beta^0 \rangle$ derived from DNS data and average slip length $\langle \beta^{int} \rangle$ calculated from experimental data at the interface. The subscript ‘S’ denotes slip boundary condition and ‘NS’ denotes no-slip velocity boundary condition.

and $\langle \beta_{NS}^0 \rangle$ with an immobilized interface. Along the actual curved liquid–liquid interface, we define and have experimental access to $\langle \beta^{int} \rangle$. All these definitions are commonly found in the literature and are used here for comparison. Ultimately, they convey the same physical information: a local measurement of the velocity at the fluid–fluid interface.

Figure 18(a) illustrates the results as a function of the groove aspect ratio and figure 18(b) as a function of the viscosity ratio. Calculating the global slip length using the spatial averages $\langle \beta_S^0 \rangle$ and $\langle \beta_{NS}^0 \rangle$ results in higher estimation than their counterparts β_S^Q and β_{NS}^Q . This discrepancy is more pronounced in the case of a surfactant-free interface and becomes increasingly significant with higher viscosity ratios and lower aspect ratios. The two approaches are equivalent in the case of a flat interface.

Retaining both effective (flow-based measurement) and spatially averaged slip length is important for comparison, as drag-reduction measurements relate to the former and local flow measurements relate to the latter.

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